HW 21 # 4

$$\lim_{x\to\infty} \frac{4x^3+9}{17x^2+11} \sim \frac{4x^3}{17x^2} = \frac{4}{17}x$$

$$\lim_{x\to\infty} \frac{4x^3+9}{17x^2+11} = \lim_{x\to\infty} \frac{4}{17}x = +\infty$$

$$\lim_{x\to-\infty} \frac{4x^3+9}{17x^2+11} = \lim_{x\to-\infty} \frac{4}{17}x = -\infty$$

Lesson 22 : a summary of curve sketching.

① $y = \dfrac{x+3}{x-4}$

Domain: all numbers x such that $y(x)$ is defined.

$(-\infty, 4)$ and $(4, +\infty)$

x-intercept: $(y = 0)$

$$\frac{x+3}{x-4} = 0 \qquad x+3 = 0$$
$$x = -3$$

$(-3, 0)$

y-intercept: $(x = 0)$

$$y(0) = \frac{0+3}{0-4} = -\frac{3}{4} \qquad (0, -3/4)$$

Behaviour at $\pm\infty$

$$\lim_{x\to\infty} \frac{x+3}{x-4} \sim \frac{x}{x} = 1$$

$$\lim_{x\to\infty} \frac{x+3}{x-4} = 1$$

$$\lim_{x\to-\infty} \frac{x+3}{x-4} = 1$$

Verticle Asy. (denominator $= 0$)

$$x - 4 = 0$$
$$x = 4$$

Horiz. Asy.

Since $\lim_{x\to\infty} \frac{x+3}{x-4} = 1$ $\left(\text{or } \lim_{x\to-\infty} \frac{x+3}{x-4} = 1\right)$,

then $y = 1$ is a horz. asy.

Slant Asy (must deg top > deg of Bot).

Since deg of $x+3$ is 1
deg of $x-4$ is 1, then no slant

Critical numbers $(y' = 0)$

$$y = \frac{x+3}{x-4} \qquad y' = \frac{(1)(x-4) - (x+3)(1)}{(x-4)^2}$$
$$= \frac{-7}{(x-4)^2}$$

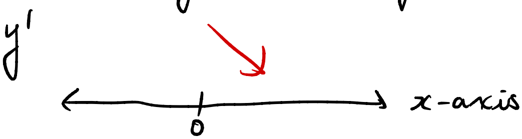
y' DNE
$x = 4$
$x = 4$ not in domain so we throw it out.

$$\frac{-7}{(x-4)^2} = 0$$
$$-7 = 0 \quad \text{this has no solutions}$$

No critical numbers.

$$y'(0) = \frac{-7}{(0-4)^2} < 0$$

Increasing / Decreasing



Increasing: nowhere
decreasing: $(-\infty, \infty)$

rel. extrema Since there are no critical numbers there are no rel. extrema.

Concavity Examine y''

$$y' = \frac{-7}{(x-4)^2} = -7(x-4)^{-2}$$

$$y'' = 14(x-4)^{-3} \cdot (1)$$

$$= \frac{14}{(x-4)^3}$$

$y'' = 0$

$$\frac{14}{(x-4)^3} = 0$$

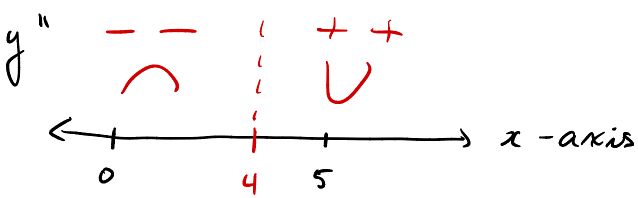
$14 = 0$

no solutions

y'' DNE

$(x-4)^3 = 0$

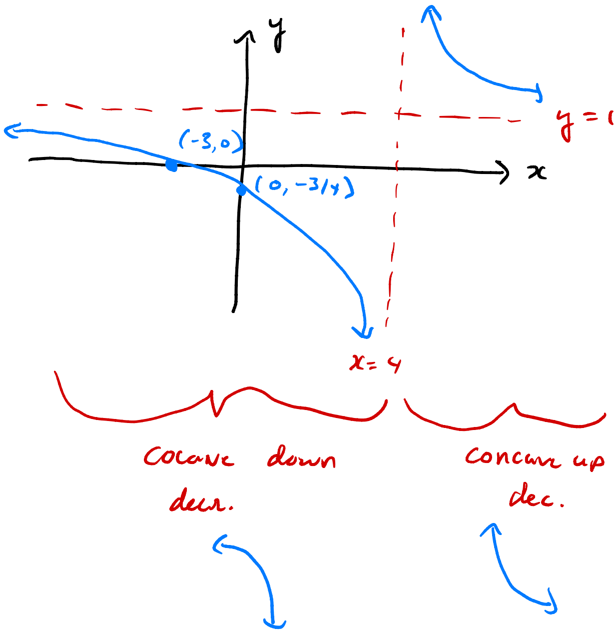
$x = 4$



$$y''(0) = \frac{14}{(0-4)^3} < 0 \qquad y''(5) = \frac{14}{(5-4)^3} > 0$$

note 4 is not an inflection point since 4 is not in domain.

Inflection points: none



② $y = \dfrac{x^2 - 3x + 9}{x - 3}$

Domain : $(-\infty, 3) \cup (3, \infty)$

x-int $(y = 0)$.

$$0 = \frac{x^2 - 3x + 9}{x - 3}$$

$$0 = x^2 - 3x + 9$$

no solutions in real numbers

y-int $(x = 0)$

$$y(0) = \frac{(0)^2 - 3(0) + 9}{0 - 3} = \frac{9}{-3} = -3$$

$(0, -3)$.

Vert. Asy (denominator = 0)

$$x - 3 = 0$$
$$x = 3$$

Horz Asy

$$\lim_{x\to\infty} \frac{x^2 - 3x + 9}{x - 3} \sim \frac{x^2}{x} = x$$

$$\lim_{x\to\infty} \frac{x^2 - 3x + 9}{x - 3} = \lim_{x\to\infty} x = +\infty$$

$$\lim_{x\to -\infty} \frac{x^2 - 3x + 9}{x - 3} = \lim_{x\to -\infty} x = -\infty$$

Since both limits are not finite, then no horz. asym.

Slant

$$y = \frac{x^2 - 3x + 9}{x - 3}$$

$$\begin{array}{r|l} & x \\ \hline x - 3 & x^2 - 3x + 9 \\ & -(x^2 - 3x) \\ \hline & \qquad 9 \end{array}$$

9 ← remainder

$$\frac{x^2}{x} = x$$

$$\cancel{\frac{9}{x}}$$

$$y = \underbrace{x}_{\text{slant}} + \frac{9}{x - 3}$$

Slant asy: $y = x$

Lesson 22: A summary of curve sketching

① $y = \dfrac{x^2 - 3x + 9}{x - 3}$

Domain: all x values such that $y(x)$ is defined.

$(-\infty, 3) \cup (3, +\infty)$

x-intercept $(y = 0)$

$$0 = \frac{x^2 - 3x + 9}{x - 3}$$

$$0 = x^2 - 3x + 9$$

$$x = \frac{3 \pm \sqrt{9 - 4 \cdot 9}}{2} = \frac{3 \pm \sqrt{-27}}{2}$$

no solutions in the domain

no x-intercepts.

y-intercepts $(x = 0)$

$$y(0) = \frac{0^2 - 3(0) + 9}{(0) - 3} = \frac{9}{-3} = -3$$

y-intercept at $(0, -3)$.

behaviour of y at $\pm\infty$

$$y = \frac{x^2 - 3x + 9}{x - 3} \sim \frac{x^2}{x} = x$$

$$\lim_{x \to \infty} \frac{x^2 - 3x + 9}{x - 3} = \lim_{x \to \infty} x = \infty, \quad \lim_{x \to -\infty} \frac{x^2 - 3x + 9}{x - 3} = \lim_{x \to -\infty} x = -\infty$$

horz asy Since neither of the above limits are finite, then no horz. asy.

Vertical asy (denominator $= 0$)

$x - 3 = 0$

$x = 3$ so we have a vert. asy. at $x = 3$.

Slant asy.

$$y = \frac{x^2 - 3x + 9}{x - 3}$$

$$\begin{array}{r|l} & x \\ \hline x - 3 & x^2 - 3x + 9 \\ & -(x^2 - 3x) \\ \hline & 9 \leftarrow \text{remainder.} \end{array}$$

$$\frac{x^2}{x} = x$$

$$\cancel{\frac{9}{x}}$$

$$y = \underbrace{x}_{\text{slant}} + \frac{9}{x - 3}$$

Slant asy: $y = x$

Critical numbers $y' = 0$ y' DNE

$$y = \frac{x^2 - 3x + 9}{x - 3}$$

$$y' = \frac{(2x - 3)(x - 3) - (x^2 - 3x + 9)(1)}{(x - 3)^2}$$

$$= \frac{2x^2 - 6x - 3x + 9 - x^2 + 3x - 9}{(x - 3)^2}$$

$$= \frac{x^2 - 6x}{(x-3)^2} = \frac{x(x-6)}{(x-3)^2}$$

$y' = 0$

$$\frac{x(x-6)}{(x-3)^2} = 0$$

$$x(x-6) = 0$$

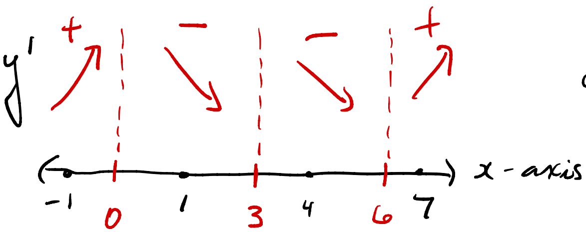
$x = 0 \quad x = 6$

y' DNE

$$(x-3)^2 = 0$$

$x = 3$ ← not crit numb.

Increasing / decreasing



Inc: $(-\infty, 0) \cup (6, \infty)$

Dec: $(0, 3) \cup (3, 6)$

$$y'(-1) = \frac{(-1)(-1-6)}{(-1-3)^2} = \frac{(-1)(-7)}{(-4)^2} > 0$$

$$y'(1) = \frac{(1)(1-6)}{(1-3)^2} < 0$$

$$y'(4) = \frac{4(4-6)}{(4-3)^2} < 0$$

$$y'(7) = \frac{7(7-6)}{(7-3)^2} > 0$$

relative ext. by above work $x=0$ is rel max and $x=6$ is rel. min.

Concavity ($y''=0$ or DNE)

$$y' = \frac{x^2-6x}{(x-3)^2}$$

$$y'' = \frac{(2x-6)(x-3)^2 - (x^2-6x)\cdot 2(x-3)}{(x-3)^4}$$

$$= \frac{(x-3)\left[(2x-6)(x-3) - 2(x^2-6x)\right]}{(x-3)^4}$$

$$= \frac{2x^2-6x-6x+18-2x^2+12x}{(x-3)^3}$$

$$= \frac{18}{(x-3)^3}$$

$y''=0$

$$\frac{18}{(x-3)^3}=0$$

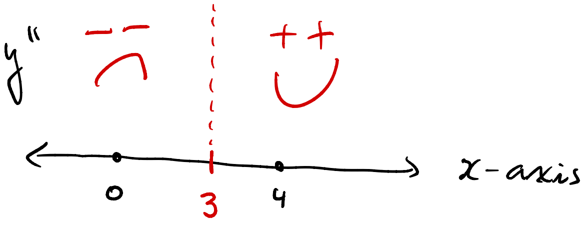
$18=0$

no solutions

y'' DNE

$(x-3)^3=0$

$x=3$



Concave up: $(3, \infty)$

Concave down: $(-\infty, 3)$

$$y''(0) = \frac{18}{(0-3)^3} < 0$$

$$y''(4) = \frac{18}{(4-3)^3} > 0$$

Inflection points

Even though concavity switches at $x = 3$, since 3 is not in the domain of y then 3 is NOT an inflection point.

Sketch