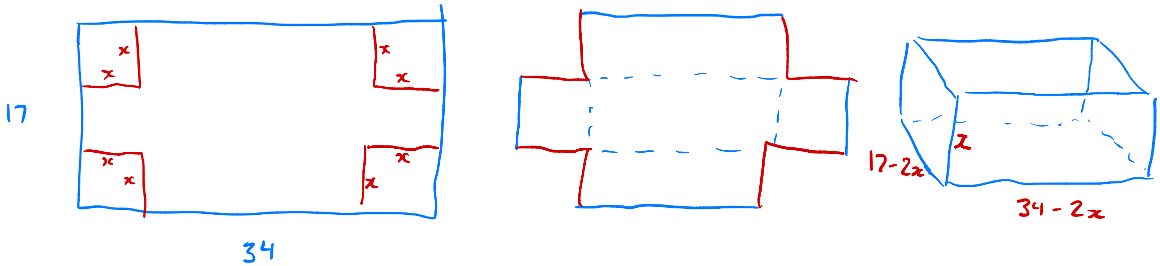
Lesson 23: Optimization I

① A piece of cardboard 17in x 34in.
Cut a square into each corner.
Fold up cardboard into a open-top box.
What is maximum volume of our box.



Objective function: The function we want to maximize (or minimize)

$V = (34-2x)(17-2x)x$

Constraint: $0 < x < 17/2$

finding the max of $V=(34-2x)(17-2x)x$ on $(0, 17/2)$

$V = (578 - 68x - 34x + 4x^2)x$

$= 578x - 102x^2 + 4x^3$

$V' = 578 - 204x + 12x^2$

$V' = 0$ or V' DNE never happens!

$0 = 12x^2 - 204x + 578$

$$x = \frac{204 \pm \sqrt{204^2 - 4(12)(578)}}{2(12)}$$

$$\approx \underbrace{13.4075}_{} \quad \text{or} \quad 3.5925$$

this is not in the interval $(0, 17/2)$

$$V' = 12x^2 - 204x + 578$$

$$V'' = 24x - 204$$

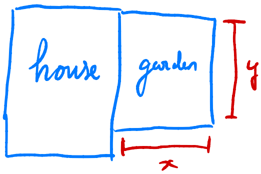
$$V''(3.5925) = 24(3.5925) - 204 < 0$$

Since second deriv. is less than zero at 3.5925 it is a rel. max.

$$V(3.5925) = (34 - 2(3.5925))(17 - 2(3.5925))(3.5925)$$

$$\approx 945.5073 \text{ in}^3$$

(2) You have 100 ft of fence to make a rectangular garden alongside the wall of your house. What is the largest possible area of your garden?



Objective function: $A = xy$

Constraint: $100 = 2x + y$, $x, y > 0$

$$y = 100 - 2x$$

$$y > 0$$
$$100 - 2x > 0$$
$$-2x > -100$$
$$x < 50$$

New obj. fun.: $A = x(100 - 2x)$

New constraint: $0 < x < 50$

We want to find max of $A = x(100-2x)$ on the interval $(0, 50)$.

$$A = 100x - 2x^2$$

$$A' = 100 - 4x$$

$A' = 0$ or A' DNE never happens!

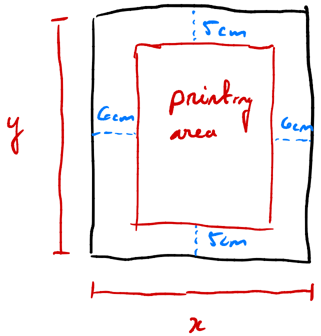
$$100 - 4x = 0$$
$$4x = 100$$
$$x = 25 \text{ ft}$$

$A'' = -4$ $\quad$ $A''(25) = -4 < 0$, so $x = 25$ is a rel max.

So we have $x = 25$ ft is the x-value of the abs. max of A on $(0, 50)$.

$$A(25) = 25\,(100 - 2(25))$$
$$= 25\,(50)$$
$$= 1250 \text{ ft}^2 \leftarrow \text{the max area of our garden.}$$

3 You are designing a poster with area 900 cm^2.



Find the maximum printing area.

Objective function : $A = (x-12)(y-10)$

Constraints : $900 = xy, \quad x, y > 0$

$$y = \frac{900}{x}$$

New obj fun. : $A = (x-12)\left(\frac{900}{x} - 10\right)$

New constraint: $12 < x < 90$

$$y = \frac{900}{x} > 10$$
$$\frac{1}{x} > \frac{10}{900} = \frac{1}{90}$$
$$x < 90$$

$$A = 900 - 10x - \frac{900 \cdot 12}{x} + 120$$
$$= 1020 - 10x - \frac{900 \cdot 12}{x}$$

$$A' = -10 + \frac{900 \cdot 12}{x^2}$$
$$= \frac{900 \cdot 12 - 10x^2}{x^2}$$

$A' = 0$ or A' DNE

$0 = 900 \cdot 12 - 10x^2$

$x^2 = 0$

$x = 0$

↑ not in interval

$10x^2 = 900 \cdot 12$

$x^2 = 90 \cdot 12$

$x = \pm\sqrt{90 \cdot 12}$ ← $-\sqrt{90 \cdot 12}$ not in interval

Only crit. num. in int. is $\sqrt{90 \cdot 12}$.

$$A'' = \frac{(-20x)(x^2) - (900 \cdot 12 - 10x^2)(2x)}{x^4}$$

$A''(\sqrt{90 \cdot 12}) < 0$, thus $x = \sqrt{90 \cdot 12}$ is rel max

$\therefore$ $\sqrt{90 \cdot 12}$ is x-value of abs. max of A on $(12, 90)$

$A(\sqrt{90 \cdot 12}) \approx 362.7329 \text{ cm}^2$ ← max area