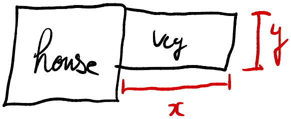
HW 23 # 4



We have 5L ft of fence

Maximize area of veg. gard.

Obj: $A = xy$

Const: $5L = 2x + y$, $x, y > 0$

$y = 5L - 2x$

$5L - 2x > 0$

Obj: $A = x(5L - 2x)$

$2x < 5L$

Const: $0 < x < \frac{5L}{2}$

$x < \frac{5L}{2}$

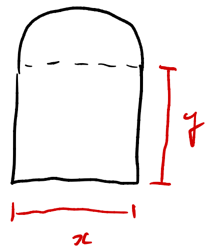
find the abs. max of $A = x(5L - 2x)$ on $\left(0, \frac{5L}{2}\right)$

$A = 5Lx - 2x^2$

$A' = 5L - 4x$

Lesson 24: Optimization II

① The following window has perimeter of 24 ft. Find dimensions of the window that allow the window to admit the most light.



Obj: (This is function we want to maximize or minimize)

$$A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$$

Const: $24 = x + 2y + \frac{1}{2}\cdot 2\pi\left(\frac{x}{2}\right)$; $x, y > 0$

$$24 = x + 2y + \frac{\pi}{2}\cdot x$$

$$2y = 24 - x - \frac{\pi}{2}x$$

$$y = 12 - \frac{1}{2}x - \frac{\pi}{4}x$$

$$= 12 - \left(\frac{1}{2} + \frac{\pi}{4}\right)x$$

$$= 12 - \frac{2+\pi}{4}x$$

New Obj: $A = x\left(12 - \frac{2+\pi}{4}x\right) + \frac{1}{8}\pi x^2$

$$= 12x - \frac{2+\pi}{4}x^2 + \frac{1}{8}\pi x^2$$

$12 - \frac{2+\pi}{4}x > 0$

New Const: $0 < x < \frac{48}{2+\pi}$

$\frac{2+\pi}{4}x < 12$

$x < \frac{48}{2+\pi}$

$$A' = 12 - \frac{2+\pi}{2}x + \frac{1}{4}\pi x$$

$A' = 0$ or A' DNE doesn't happen here

$$12 - \frac{2+\pi}{2}x + \frac{1}{4}\pi x = 0$$

$$12 + x\left(-\frac{2+\pi}{2} + \frac{\pi}{4}\right) = 0$$

$$12 + x\left(\frac{-4-2\pi+\pi}{4}\right) = 0$$

$$12 + x\left(\frac{-4-\pi}{4}\right) = 0$$

$$-x\left(\frac{4+\pi}{4}\right) = -12$$

$$x = \frac{48}{4+\pi}$$ ← this is in $\left(0, \frac{48}{2+\pi}\right)$

$$A' = 12 - \frac{2+\pi}{2}x + \frac{1}{4}\pi x$$

$$A'' = 0 - \frac{2+\pi}{2} + \frac{1}{4}\pi$$

$$= \frac{-4-2\pi+\pi}{4}$$

$$= -\frac{(4+\pi)}{4} < 0$$

$A''\left(\frac{48}{4+\pi}\right) = -\frac{(4+\pi)}{4} < 0$, so $x = \frac{48}{4+\pi}$ is a rel max.

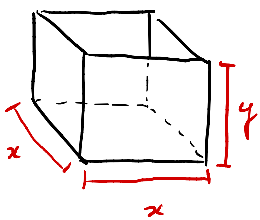
$x = \frac{48}{4+\pi}$ is an abs max on $\left(0, \frac{48}{2+\pi}\right)$

$y = 12 - \frac{2+\pi}{4}x$ $\qquad y\left(\frac{48}{4+\pi}\right) = 12 - \frac{2+\pi}{4} \cdot \frac{48}{4+\pi}$

dimensions are $x = \frac{48}{4+\pi}$ ft $\quad y = 12 - \frac{2+\pi}{4} \cdot \frac{48}{4+\pi}$ ft.

(2) A box w/ a square base, no top, has volume 6912 in^3

Find the dim. of box that require the least amount of material.



Obj: $S = x^2 + 4xy$

Const: $6912 = x^2 y$; $x, y > 0$

$$y = \frac{6912}{x^2}$$

New obj: $S = x^2 + 4x\left(\frac{6912}{x^2}\right) = x^2 + \frac{4 \cdot 6912}{x}$

Const: $0 < x < +\infty$

$$\frac{6912}{x^2} > 0$$

always true $x \neq 0$

find abs. min of $S = x^2 + \frac{4 \cdot 6912}{x}$ on $(0, \infty)$.

$$S' = 2x - \frac{4 \cdot 6912}{x^2}$$

$$= \frac{2x^3 - 4 \cdot 6912}{x^2}$$

$S' = 0$ or S' DNE

$2x^3 - 4 \cdot 6912 = 0$

$x^3 = 2 \cdot 6912$

$x = \sqrt[3]{2 \cdot 6912} = 24$

$x^2 = 0$

$x = 0$ ← this is not in $(0, \infty)$ so ignore it.

$$S' = \frac{2x^3 - 4 \cdot 6912}{x^2}$$

$$S'' = \frac{(6x^2)(x^2) - (2x^3 - 4 \cdot 6912)(2x)}{x^4}$$

$$= \frac{6x^4 - 4x^4 + 8 \cdot 6912x}{x^4}$$

$$= \frac{6x^3 - 4x^3 + 8 \cdot 6912}{x^3}$$

$$= \frac{2x^3 + 8 \cdot 6912}{x^3}$$

$$S''(24) = \frac{2(24)^3 + 8 \cdot 6912}{(24)^3} > 0$$

so $x = 24$ is a rel min.

so $x = 24$ is an abs min of S on $(0, \infty)$

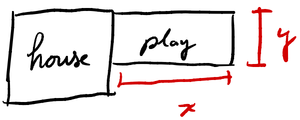
$$y = \frac{6912}{x^2} \qquad y(24) = \frac{6912}{(24)^2}$$

Our dimensions of the box are

$x = 24$ in and $y = \frac{6912}{24^2}$ in give volume 6912 in^3

and they minimize the amount of material used.

HW 23 # 5



area of play area is $2500\ ft^2$

minimize the amount of fence

Obj: $P = x + y + x = 2x + y$

Const: $2500 = xy$; $x, y > 0$.

$$y = \frac{2500}{x}$$

New obj: $P = 2x + \dfrac{2500}{x}$

New const: $0 < x < \infty$

$$\frac{2500}{x} > 0$$

$$\frac{1}{x} > 0$$

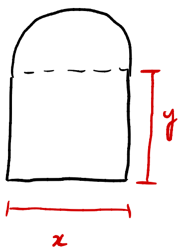
$$x > 0$$

We want to find the abs. min of

$$P = 2x + \frac{2500}{x} \quad \text{on} \quad (0, \infty)$$

Lesson 24: Optimization II

① The following window has perimeter 24 ft. Find the dimensions of the window that allow it admit the most light.



Obj: (This is function we seek to minimize or maximize)

$$A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$$

Const: $24 = x + 2y + \frac{1}{2}\cdot 2\pi\left(\frac{x}{2}\right)$; $x, y > 0$

$$2y = 24 - x - \frac{\pi}{2}x$$

$$y = 12 - \frac{1}{2}x - \frac{\pi}{4}x$$

$$= 12 - x\left(\frac{1}{2} + \frac{\pi}{4}\right)$$

$$y = 12 - x\cdot\frac{2+\pi}{4}$$

New obj: $A = x\left(12 - x\cdot\frac{2+\pi}{4}\right) + \frac{\pi}{8}x^2$

$$= 12x - x^2\cdot\frac{2+\pi}{4} + \frac{\pi}{8}x^2$$

$$12 - x\cdot\frac{2+\pi}{4} > 0$$

$$x\cdot\frac{2+\pi}{4} < 12$$

$$x < \frac{48}{2+\pi}$$

New const: $0 < x < \frac{48}{2+\pi}$

find the abs. max of A on $\left(0, \frac{48}{2+\pi}\right)$.

$$A' = 12 - x\cdot\frac{2+\pi}{2} + \frac{\pi}{4}x$$

$A' = 0$ or A' DNE doesn't here

$$12 - x \cdot \frac{2+\pi}{2} + \frac{\pi}{4} x = 0$$

$$12 + x\left(-\frac{2+\pi}{2} + \frac{\pi}{4}\right) = 0$$

$$12 + x \cdot \frac{-4 - 2\pi + \pi}{4} = 0$$

$$12 - x \cdot \frac{4+\pi}{4} = 0$$

$$x \cdot \frac{4+\pi}{4} = 12$$

$$x = \frac{48}{4+\pi} \leftarrow$$ this is in fact inside $\left(0, \frac{48}{2+\pi}\right)$ ✓

$$A' = 12 - x \cdot \frac{2+\pi}{2} + \frac{\pi}{4} x$$

$$A'' = 0 - \frac{2+\pi}{2} + \frac{\pi}{4} = \frac{-4 - 2\pi + \pi}{4} = \frac{-(4+\pi)}{4}$$

$A''\left(\frac{48}{4+\pi}\right) = -\frac{(4+\pi)}{4} < 0$, so $x = \frac{48}{4+\pi}$ is a rel max.

So $x = \frac{48}{4+\pi}$ is an abs. max on $\left(0, \frac{48}{2+\pi}\right)$

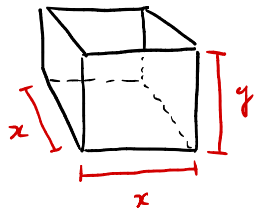
$$y = 12 - x \cdot \frac{2+\pi}{4} \qquad y\left(\frac{48}{4+\pi}\right) = 12 - \left(\frac{48}{4+\pi}\right)\left(\frac{2+\pi}{4}\right)$$

$$= 12 - \frac{12(2+\pi)}{4+\pi}$$

So dim. are $x = \frac{48}{4+\pi}$ ft and $y = 12 - \frac{12(2+\pi)}{4+\pi}$ ft.

② A box w/ square base, no top, volume of 6912 in^3.

Find the dimensions of the box that require the least ammount of material.



Obj: $S = x^2 + 4xy$

Const: $6912 = x^2 y$; $x, y > 0$

$$y = \frac{6912}{x^2}$$

New obj: $S = x^2 + 4x\left(\frac{6912}{x^2}\right) = x^2 + \frac{4 \cdot 6912}{x}$

New const: $0 < x < +\infty$

$\frac{6912}{x^2} > 0$

$\frac{1}{x^2} > 0$

always true for $x \neq 0$.

find the abs. min of $S = x^2 + \frac{4 \cdot 6912}{x}$ on $(0, \infty)$.

$$S' = 2x - \frac{4 \cdot 6912}{x^2} = \frac{2x^3 - 4 \cdot 6912}{x^2}$$

$S' = 0$ or S' DNE

$2x^3 - 4 \cdot 6912 = 0$

$x^3 = 2 \cdot 6912$

$x = \sqrt[3]{2 \cdot 6912}$

$= 24$

$x^2 = 0$

$x = 0$ ← this is not in $(0, \infty)$ so ignore it.

$$S' = \frac{2x^3 - 4 \cdot 6912}{x^2}$$

$$S'' = \frac{(6x^2)(x^2) - (2x^3 - 4 \cdot 6912)(2x)}{x^4}$$

$$= \frac{6x^4 - 4x^4 + 8 \cdot 6912x}{x^4}$$

$$= \frac{6x^3 - 4x^3 + 8 \cdot 6912}{x^3}$$

$$= \frac{2x^3 + 8 \cdot 6912}{x^3}$$

$S''(24) = \dfrac{2(24)^3 + 8 \cdot 6912}{(24)^3} > 0$ so $x = 24$ is a rel min

So $x = 24$ is an abs. min on $(0, \infty)$

Dim. of box: $x = 24$ in

$$y = \frac{6912}{(24)^2} = \frac{6912}{24^2} \text{ in}$$

How much material is required at minimum?

$$S = x^2 + 4xy$$

$$S = 24^2 + 4(24) \cdot \frac{6912}{(24)^2} = 24^2 + \frac{4 \cdot 6912}{24} \text{ in}^2$$