Lesson 25: Optimization III

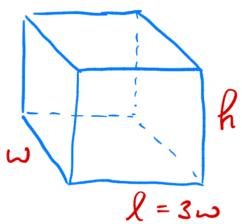
① Construct a box with volume of 35 ft^3 using metal and wood.

metal cost: $\$14 / \text{ft}^2$

wood cost: $\$8 / \text{ft}^2$

- Put wood on sides, put metal on top / bot.
- Length of the base is 3 times the width of base.

Find the dim. of box minimize cost of construction.



$l = 3 \cdot w$

$$35 = lwh$$
$$= 3w^2 h$$

Obj function: $C = 14(2 \cdot 3w \cdot w) + 8(3wh + 3wh + wh + wh)$

$$= 6 \cdot 14 w^2 + 8(8wh)$$
$$= 84w^2 + 64wh$$

Constraint: $35 = 3w^2 h \quad ; \quad w, h > 0$

$$h = \frac{35}{3w^2}$$

New obj: $C = 84w^2 + 64w\left(\frac{35}{3w^2}\right) = 84w^2 + \frac{2240}{3w}$

New constraint : $0 < w < +\infty$

$\frac{35}{3w^2} > 0$
always true for $w \neq 0$

find the abs. min. of $C = 84w^2 + \frac{2240}{3w}$
on the interval $(0, \infty)$.

↑ solve this

(2) • If a product is sold by a company at p dollars per unit, then they will sell $q = 2800 - 100p$ units.

• Each unit costs \$3 to make.

a) What price should the company charge to maximize revenue?

Revenue = amount of money generated.

= (price of item)(number of items sold)

$= (p)(2800 - 100p)$

$R = 2800p - 100p^2$

obj fun: $R = 2800p - 100p^2$
Constraint: $0 < p < \infty$
find the abs. max of $R = 2800p - 100p^2$ on $(0, \infty)$.

b) What price should the company charge to maximize profit.

Profit = Revenue − Cost of production

$= 2800p - 100p^2 -$ (cost per unit)(number of units)

Obj fun: $P = 2800p - 100p^2 - (3)(2800 - 100p)$

Const: $0 < p < \infty$

maximize this function.

Extra Problem!! ↓↓↓↓↓↓

(3) Find the points on curve $y = x^2 + 1$ closest to the point $(0,5)$.

Equation for distance between 2 points (x, y) and (x_0, y_0) is

$$d^2 = (x - x_0)^2 + (y - y_0)^2$$

Obj fun: $D = (x - 0)^2 + (y - 5)^2 = x^2 + (y-5)^2$

Const: $y = x^2 + 1$

solve for x^2, then $x^2 = y - 1$

New obj : $\mathcal{D} = y - 1 + (y-5)^2$

$$= y - 1 + y^2 - 10y + 25$$

$$= y^2 - 9y + 24$$

New const : $-\infty \le y \le \infty$

find abs. min of $\mathcal{D} = y^2 - 9y + 24$ on $(-\infty, \infty)$.

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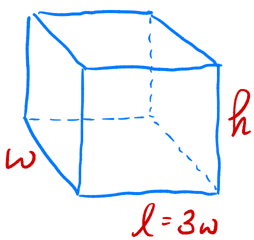
Find the dim. of the box that minimize the cost of construction.



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$$= 14(6w^2) + 8(8wh)$$

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Constraint: $35 = w(3w)h = 3w^2h; \quad w, h > 0$

$$h = \frac{35}{3w^2}$$

New obj: $C = 84w^2 + 64w\left(\frac{35}{3w^2}\right)$

$= 84w^2 + \frac{2240}{3w}$

$\frac{35}{3w^2} > 0$
this is always true for $w \neq 0$.

New const: $0 < w < +\infty$

find the abs. minimum of

$C = 84w^2 + \frac{2240}{3w}$ on the interval $(0, \infty)$.

(2) If a company sells a product at p dollars per unit they will sell $q = 2800 - 100p$ units.

- Each unit costs \$3 to make

a) What price should the company charge to maximize revenue?

Revenue = (price per unit)(number of units sold)

$R = (p)(2800 - 100p)$

$= 2800p - 100p^2$

Obj: $R = 2800p - 100p^2$

Const: $0 < p < 28$

find the abs. max of $R = 2800p - 100p^2$ on $(0, \infty)$.

b) What price should the company charge to max. profit.

Profit = Revenue − (cost per unit)(number of units sold)

$$P = 2800p - 100p^2 - (3)(2800 - 100p)$$

Obj fun: $P = 2800p - 100p^2 - 3(2800 - 100p)$

Const: $0 < p < 28$

find abs. max of P on $(0, 28)$.