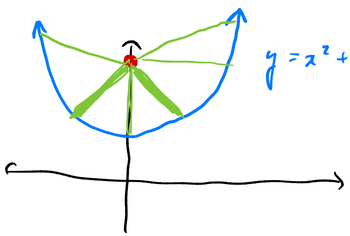
HW 25 # 5 | Find pts on $y = x^2 + 1$ closest to the pt $(0,5)$.



Eq dist b/w (x, y) and (x_0, y_0)

$$d^2 = (x - x_0)^2 + (y - y_0)^2$$

d is dist b/w the pts.

Obj: $D = (x-0)^2 + (y-5)^2 = x^2 + y^2 - 10y + 25$

Const: $y = x^2 + 1$

$x^2 = y - 1$

$D = y - 1 + y^2 - 10y + 25 = y^2 - 9y + 24$

$D' = 2y - 9$

$D' = 0$ $\qquad$ D' DNE doesn't happen

$2y = 9$

$y = 9/2$

$D'' = 2$ $\qquad$ $D''(9/2) = 2 > 0$ so $y = 9/2$ abs min

Lesson 26: Antiderivatives and indefinite integration

Antiderivatives

$f(x) = 3x^2$. Is there a function $F(x)$ such that $F'(x) = 3x^2$?

Yes! $F(x) = x^3$, x^3+1, x^3-100

Def. A function $F(x)$ is an antiderivative of $f(x)$ if $F'(x) = f(x)$.

Antiderivatives are not unique, they differ by a constant.

Indefinite integration

Let $F(x)$ be an antiderivative of $f(x)$, then define the indefinite integral of $f(x)$ as

$$\int f(x)\,dx := F(x) + C$$

integral sign

integrand

integration variable

constant of integration

Basic integration rules

$$\int 0 \, dx = c$$

$$\int k \, dx = kx + c$$

$$\int k f(x) \, dx = k \int f(x) \, dx$$

$$\int f(x) + g(x) \, dx = \int f(x) \, dx + \int g(x) \, dx$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \qquad n \neq -1$$

$$\frac{\cancel{(n+1)} x^n}{\cancel{(n+1)}} + 0$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \csc^2 x \, dx = -\cot x + c$$

$$\int \sec x \tan x \, dx = \sec x + c$$

$$\int \csc x \cot x \, dx = -\csc x + c$$

LON-CAPA

abs(x)

$$\int e^x \, dx = e^x + c \qquad \int \frac{1}{x} \, dx = \ln|x| + c$$

Examples

① $\displaystyle\int 6x^2 + \frac{\sqrt[3]{x^2}}{6}\,dx$

$$= \int 6x^2\,dx + \int \frac{\sqrt[3]{x^2}}{6}\,dx$$

$$= 6\int x^2\,dx + \frac{1}{6}\int \sqrt[3]{x^2}\,dx$$

$$= 6\int x^2\,dx + \frac{1}{6}\int x^{2/3}\,dx$$

$$= 6\,\frac{x^{2+1}}{2+1} + \frac{1}{6}\cdot\frac{x^{2/3+1}}{\frac{2}{3}+1} + C$$

$$= \frac{6}{3}x^3 + \frac{1}{6}\cdot\frac{x^{5/3}}{5/3} + C$$

$$= 2x^3 + \frac{1}{6}\cdot\frac{3}{5}x^{5/3} + C$$

$$= 2x^3 + \frac{1}{10}x^{5/3} + C$$

Check: derivative $= 6x^2 + \frac{1}{10}\cdot\frac{5}{3}x^{5/3-1} + 0$

$= 6x^2 + \frac{1}{6}x^{2/3}$ ✓

② $\int 2 \tan x \cos x + 3 \, dx$ $\qquad \tan x = \dfrac{\sin x}{\cos x}$

$$= \int 2 \sin x + 3 \, dx$$

$$= \int 2 \sin x \, dx + \int 3 \, dx$$

$$= 2 \int \sin x \, dx + 3 \int 1 \, dx$$

$$= -2 \cos x + 3x + C$$

③ $\int \dfrac{1 + 7xe^x}{x} dx = \int \dfrac{1}{x} + \dfrac{7xe^x}{x} dx$

$$= \int \frac{1}{x} + 7e^x \, dx$$

$$= \int \frac{1}{x} dx + 7 \int e^x \, dx$$

$$= \ln|x| + 7e^x + C$$

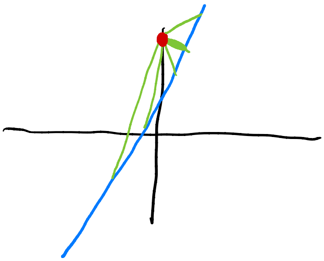
④ $\int \sec x (\sec x + 3 \tan x) \, dx$

$$= \int \sec^2 x + 3 \sec x \tan x \, dx$$

$$= \int \sec^2 x \, dx + 3 \int \sec x \tan x \, dx$$

$$= \tan x + 3 \sec x + C$$

HW 25 #4 | Find the pts on curve $y = 4x + 1$ closest to pt. $(0, 4)$.



Eq for dist. b/w pts (x, y) and (x_0, y_0)

$$d^2 = (x - x_0)^2 + (y - y_0)^2$$

where d is the distance b/w the pts.

Obj: $\mathcal{D} = (x-0)^2 + (y-4)^2$

$$= x^2 + y^2 - 8y + 16$$

Const: $y = 4x + 1$

$$\mathcal{D} = x^2 + (4x+1)^2 - 8(4x+1) + 16$$

$$= x^2 + 16x^2 + 8x + 1 - 32x - 8 + 16$$

$$= 17x^2 - 24x + 9$$

Find the abs. min of $\mathcal{D} = 17x^2 - 24x + 9$ on interval $(-\infty, \infty)$.

Lesson 26: Antiderivatives and indefinite integration

Antiderivatives

$f(x) = 3x^2$. Is there a function $F(x)$ such that $F'(x) = 3x^2$?

Yes! $F(x) = x^3, \quad x^3 + 1, \quad x^3 - 100 + \pi^2$

Def. A function $F(x)$ is an antiderivative of $f(x)$ if $F'(x) = f(x)$.

* Antiderivatives are not unique, they differ by a constant.

Indefinite integration

Let $F(x)$ be an antiderivative of $f(x)$, then we define the indefinite integral of $f(x)$ as

$$\int f(x)\, dx := F(x) + C$$

integral sign

integrand

integration variable

constant of integration

Basic rules of integration

$$\int 0 \, dx = c$$

$$\int k \, dx = kx + c$$

$$\int k f(x) \, dx = k \int f(x) \, dx$$

$$\int f(x) + g(x) \, dx = \int f(x) \, dx + \int g(x) \, dx$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \qquad \underline{n \neq -1}$$

$$\cancel{\frac{n+1}{n+1}} x^n + 0$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \csc^2 x \, dx = -\cot x + c$$

$$\int \sec x \tan x \, dx = \sec x + c$$

$$\int \csc x \cot x \, dx = -\csc x + c$$

$$\int e^x \, dx = e^x + c \qquad \int \frac{1}{x} \, dx = \ln|x| + c$$

LON-CAPA

$|x| = \text{abs}(x)$

Examples

(1) $\int 6x^2 + \frac{\sqrt[3]{x^2}}{6}\, dx$

$= \int 6x^2\, dx + \int \frac{\sqrt[3]{x^2}}{6}\, dx$

$= 6\int x^2\, dx + \frac{1}{6}\int \sqrt[3]{x^2}\, dx$

$= 6\int x^2\, dx + \frac{1}{6}\int x^{2/3}\, dx$

$= 6 \cdot \frac{x^{2+1}}{2+1} + \frac{1}{6} \cdot \frac{x^{2/3+1}}{\frac{2}{3}+1} + C$

$= \frac{6}{3} x^3 + \frac{1}{6} \cdot \frac{3}{5} x^{5/3} + C$

$= 2x^3 + \frac{1}{10} x^{5/3} + C$

derivative : $6x^2 + \frac{1}{10} \cdot \frac{5}{3} x^{2/3} + 0$

$6x^2 + \frac{1}{6} \sqrt[3]{x^2}$

(2) $\int 2 \tan x \cos x + 3\, dx$ $\qquad \tan x = \frac{\sin x}{\cos x}$

$= \int 2 \sin x + 3\, dx$

$= 2 \int \sin x\, dx + 3 \int 1\, dx$

$= -2\cos x + 3x + C$ ✓

(3) $\displaystyle\int \frac{1 + 7xe^x}{x}\,dx$

$$= \int \frac{1}{x} + \frac{7xe^x}{x}\,dx$$

$$= \int \frac{1}{x} + 7e^x\,dx$$

$$= \int \frac{1}{x}\,dx + 7\int e^x\,dx$$

$$= \ln|x| + 7e^x + C \quad \checkmark$$

(4) $\displaystyle\int \sec x\,(\sec x + 3\tan x)\,dx$

$$= \int \sec^2 x + 3\sec x \tan x\,dx$$

$$= \int \sec^2 x\,dx + 3\int \sec x \tan x\,dx$$

$$= \tan x + 3\sec x + C \quad \checkmark$$