# Lesson 27: Antiderivatives and indefinite integration II

Recall:

**Def.** A function $F(x)$ is an antiderivative of $f(x)$ if $F'(x) = f(x)$.

We define the indefinite integral of $f(x)$ as

$$\int f(x)\,dx := F(x) + C.$$

## Intial value ordinary differential equations (ODE's)

① Let $f'(x) = 15x^2 + 8x$ and $f(2) = 9$.
Find $f(-1) = ?$

$$f(x) = \int f'(x)\,dx = \int 15x^2 + 8x\,dx$$

$$= \int 15x^2\,dx + \int 8x\,dx$$

$$= 15\int x^2\,dx + 8\int x^1\,dx$$

$$= 15\cdot\frac{x^{2+1}}{2+1} + 8\cdot\frac{x^{1+1}}{1+1} + C$$

$$= 5x^3 + 4x^2 + C$$

Check: $15x^2 + 8x + 0$ matches ✓

Intial value: $f(2) = 9$ $\quad f(x) = 5x^3 + 4x^2 + C$

$$9 = f(2) = 5(2)^3 + 4(2)^2 + c$$

$$9 = 40 + 16 + c$$

$$C = -47$$

$$f(x) = 5x^3 + 4x^2 - 47$$

$$f(-1) = 5(-1)^3 + 4(-1)^2 - 47$$

$$= -5 + 4 - 47$$

$$= -48$$

(2) let $y'' = 3e^x + 4$ and $y'(0) = 8$, $y(2) = 3e^2$

Find $y(3) = ?$

$$y' = \int y''\,dx = \int 3e^x + 4\,dx$$

$$= \int 3e^x\,dx + \int 4\,dx$$

$$= 3\int e^x\,dx + 4\int 1\,dx$$

$$y' = 3e^x + 4x + C_1$$

$$8 = y'(0) = 3e^0 + 4(0) + C_1$$

$$8 = 3 + c_1$$

$$c_1 = 5$$

So $y' = 3e^x + 4x + 5$

$$y = \int y' \, dx = \int 3e^x + 4x + 5 \, dx$$

$$= \int 3e^x \, dx + \int 4x \, dx + \int 5 \, dx$$

$$= 3\int e^x \, dx + 4\int x^1 \, dx + 5\int dx$$

$$= 3e^x + 4 \cdot \frac{x^{1+1}}{1+1} + 5x + C_2$$

$$y = 3e^x + 2x^2 + 5x + c_2$$

Recall : $y(2) = 3e^2$

$$3e^2 = y(2) = 3e^2 + 2(2)^2 + 5(2) + c_2$$

$$c_2 = -(8 + 10) = -18$$

$$y = 3e^x + 2x^2 + 5x - 18$$

$$y(3) = 3e^3 + 2(3)^2 + 5(3) - 18$$

$$= 3e^3 + 15$$

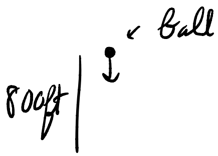
Physics ideas:

acceleration = derivative of velocity $a = s''$

velocity = derivative of position $v = s'$

velocity = integral of acceleration

position = integral of velocity.




$\overbrace{\text{intial velocity}}^{t=0}$ of 2 ft/sec

Use acceleration due to gravity to be

$a(t) = -32 \text{ ft/sec}^2$

find velocity function of the ball.

$$v(t) = \int a(t)\,dt = \int -32\,dt = -32t + C$$

$$2 = v(0) = -32(0) + C$$

$$C = 2$$

# Lesson 27: Antiderivatives and indefinite integrals II

Recall

**Def.** A function $F(x)$ is <u>an</u> **antiderivative** of $f(x)$ if

$$F'(x) = f(x)$$

We define the **indefinite integral** of $f(x)$ as

$$\int f(x)\,dx = F(x) + C$$

**Idea:** given $f''(x)$, then

- $f'(x) = \int f''(x)\,dx$
- $f(x) = \int f'(x)\,dx$

## Initial value ordinary differential equations (ODE)

(1) let $\underbrace{f''(x) = 3e^x + 4}_{\text{ODE}}$ and $\underbrace{f'(0) = 8,\ f(2) = 3e^2}_{\text{intial values.}}$.

Find $f(3)$.

$$f'(x) = \int f''(x)\,dx = \int 3e^x + 4\,dx$$

$$= \int 3e^x\,dx + \int 4\,dx$$

$$= 3\int e^x\,dx + 4\int 1\,dx$$

$f'(x) = 3e^x + 4x + C_1 \qquad f'(0) = 8$

$8 = f'(0) = 3e^0 + 4(0) + C_1$

$8 = 3 + C_1$

$C_1 = 5$

So $f'(x) = 3e^x + 4x + 5$

$f(x) = \int f'(x)\,dx = \int 3e^x + 4x + 5\,dx$

$= \int 3e^x\,dx + \int 4x\,dx + \int 5\,dx$

$= 3\int e^x\,dx + 4\int x^1\,dx + 5\int dx$

$= 3e^x + 4 \cdot \frac{x^{1+1}}{1+1} + 5x + C_2$

$f(x) = 3e^x + 2x^2 + 5x + C_2, \quad f(2) = 3e^2$

$3e^2 = f(2) = 3e^2 + 2(2)^2 + 5(2) + C_2$

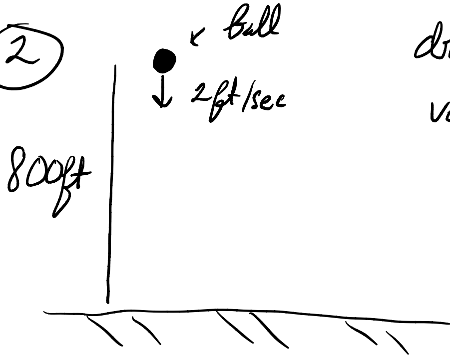
$C_2 = 3e^2 - 3e^2 - 2(2)^2 - 5(2) = -18$

$f(x) = 3e^x + 2x^2 + 5x - 18$

$f(3) = 3e^3 + 2(3)^2 + 5(3) - 18 = 3e^3 + 15$ ✓

$f(-1) = 3e^{-1} + 2(-1)^2 + 5(-1) - 18 = 3e^{-1} - 21$

(2)




dropped with an intial velocity of 2 ft/sec

Use $a(t) = -32\ ft/sec^2$ for acceleration due to gravity.

a) How long will it take for the ball to hit the ground?

- acceleration is the second derivative of position
- Velocity is the integral of acceleration
- position is the integral of velocity.

position function of the ball $s(t)$ tells us how high the ball is.

$a(t) = -32$ we want to find $v(t)$.

$$v(t) = \int a(t)\, dt = \int -32\, dt = -32 \int dt = -32t + C_1$$

$$-2 = v(0) = -32(0) + C_1$$

$$-2 = C_1$$

$$v(t) = -32t - 2$$

$$S(t) = \int v(t)\,dt = \int -32t - 2\,dt$$

$$= -32 \int t\,dt - 2\int dt$$

$$= -32\,\frac{t^{1+1}}{1+1} - 2t + C_2$$

$$= -16t^2 - 2t + C_2 \qquad S(0) = 800$$

$$800 = -16(0)^2 - 2(0) + C_2$$

$$C_2 = 800$$

$$S(t) = -16t^2 - 2t + 800$$

In order to find when the ball hits the ground $0 = -16t^2 - 2t + 800$.