HW 27 #4 $\quad y' = 5\cos x + 2 \qquad y\left(\frac{3\pi}{2}\right) = 9$

find y.

$$y = \int y' \, dx$$
$$= \int 5\cos x + 2 \, dx$$
$$= 5\int \cos x \, dx + 2\int dx$$
$$y = 5\sin x + 2x + C$$

$$9 = y\left(\frac{3\pi}{2}\right) = 5\sin\left(\frac{3\pi}{2}\right) + 2\left(\frac{3\pi}{2}\right) + C$$

$$9 = -5 + 3\pi + C$$

$$C = 14 - 3\pi$$

$$y = 5\sin x + 2x + 14 - 3\pi$$

Lesson 28: Area and Riemann sums

Sigma notation

The sigma notation allows us to concisely write all of the terms in a sum

$$1+2+3+4+5+\cdots+95+96+97+98+99+100$$

$$=\sum_{i=1}^{100} i$$

← ending value for i

i = index

← starting value for i

rule

$$\sum_{i=1}^{5} i^2 = \overset{i=1}{1^2} + \overset{i=2}{2^2} + \overset{i=3}{3^2} + \overset{i=4}{4^2} + \overset{i=5}{5^2}$$

① Write $\sum_{i=3}^{6} i(\sqrt{i}+4)$ in addition.

$$\sum_{i=3}^{6} i(\sqrt{i}+4) = \overset{i=3}{3(\sqrt{3}+4)} + \overset{i=4}{4(\sqrt{4}+4)} + \overset{i=5}{5(\sqrt{5}+4)}$$

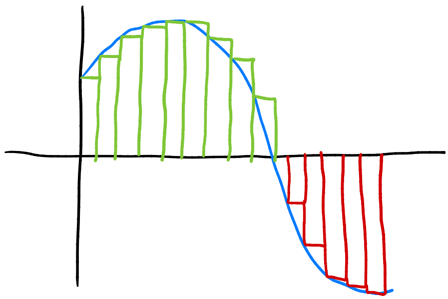
$$+ \overset{i=6}{6(\sqrt{6}+4)}$$

② Write $(1-1)^3+(2-1)^3+(3-1)^3+\cdots+(n-1)^3$

n is a positive integer.

$$\sum_{i=1}^{n}(i-1)^3 = \overset{i=1}{(1-1)^3} + \overset{i=2}{(2-1)^3} + \cdots + \overset{i=n}{(n-1)^3}$$ ✓

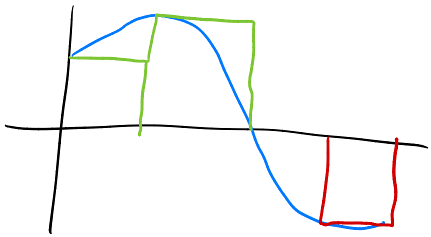
Riemann sums



Signed area under the curve is approximately

$\sum$ area of green rectangles (above x-axis)

$-\sum$ area of red rectangles (below x-axis)

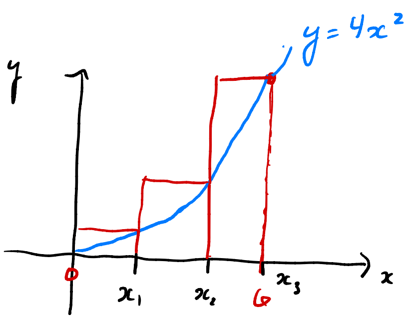


Bad approximation

So we want to use a lot of rectangles.

(3) Approximate signed area under the curve $y = 4x^2$ on $[0, 6]$ using 3 rectangles.

Right Riemann sum (rectangles right top corner touches the curve)



$\Delta x :=$ width of each rectangle

$= \dfrac{6-0}{3} = 2$

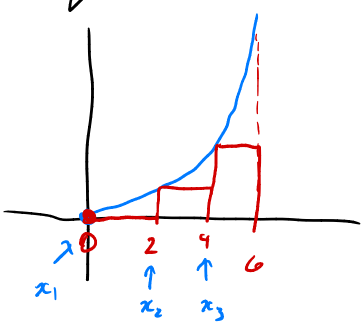
$$\sum_{i=1}^{3} y(x_i)\,\Delta x = \sum_{i=1}^{3} y(0+i\Delta x)\,\Delta x$$

$$= \sum_{i=1}^{3} 4(i\cdot 2)^2\cdot 2$$

$$= \overset{i=1}{4(1\cdot 2)^2\cdot 2} + \overset{i=2}{4(2\cdot 2)^2\cdot 2} + \overset{i=3}{4(3\cdot 2)^2\cdot 2}$$

$$= \underbrace{4(2)^2\cdot 2}_{\text{area of 1st rect.}} + \underbrace{4(4)^2\cdot 2}_{\text{area of second rect.}} + \underbrace{4(6)^2\cdot 2}_{\text{area of third rect.}}$$

Left Riemann sums



$$\Delta x = \frac{6-0}{3} = 2$$

$$\sum_{i=0}^{2} y(x_i)\,\Delta x$$

$$= \sum_{i=0}^{2} y(0+i\Delta x)\,\Delta x$$

$$= \sum_{i=0}^{2} 4(i\cdot 2)^2\,\Delta x$$

$$= \underset{i=0}{4(0\cdot 2)^2\cdot 2} + \underset{i=1}{4(1\cdot 2)^2\cdot 2} + \underset{i=2}{4(2\cdot 2)^2\cdot 2}$$

General Case $f(x)$ on $[a, b]$ w/ n rectangles.

Right Riemann sums:

$$\sum_{i=1}^{n} f(a + i\Delta x)\Delta x \qquad \Delta x = \frac{b-a}{n}$$

left Riemann Sum

$$\sum_{i=0}^{n-1} f(a + i\Delta x)\Delta x$$

HW 27 #7 growth rate of a pop. of Bacteria, $\frac{dP}{dt}$, is proportional to $5\sqrt{t}$.

P : population

t : time in days $(0 \le t \le 10)$

Intial pop is 300

Approximate pop after 7 days.

$\frac{dP}{dt} = 5\sqrt{t}$, $P(0) = 300$, Find $P(7) = ?$

$$P = \int \frac{dP}{dt}\, dt = \int 5\sqrt{t}\, dt = 5\int t^{1/2}\, dt$$

$$= 5\frac{t^{3/2}}{3/2} + C$$

$$= 5 \cdot \frac{2}{3} t^{3/2} + C$$

$$= \frac{10}{3} t^{3/2} + C$$

$$300 = P(0) = \frac{10}{3}(0)^{3/2} + C$$

$$C = 300$$

$$P(t) = \frac{10}{3} t^{3/2} + 300$$

$P(7) = \frac{10}{3}(7)^{3/2} + 300 \approx 362$ pop after 7 days

Lesson 28: Area and Riemann sums

Sigma notation

The **sigma (sum) notation** alows us to concisely write all of the terms in a sum

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2$$

ending value → 10 ↓ rule

$$\sum_{i=1}^{10} i^2 = \underbrace{1^2}_{i=1} + \underbrace{2^2}_{i=2} + \underbrace{3^2}_{i=3} + \cdots + \underbrace{9^2}_{i=9} + \underbrace{10^2}_{i=10}$$

index → $i=1$

↑ starting value

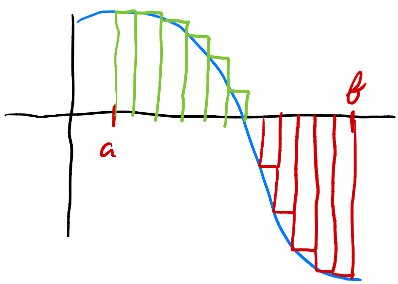
① Write $\sum_{i=3}^{6} i(\sqrt{i}+4)$ in addition.

$$\sum_{i=3}^{6} i(\sqrt{i}+4) = \underbrace{3(\sqrt{3}+4)}_{i=3} + \underbrace{4(\sqrt{4}+4)}_{i=4} + \underbrace{5(\sqrt{5}+4)}_{i=5} + \underbrace{6(\sqrt{6}+4)}_{i=6}$$

② Write $\underbrace{(1-1)^3}_{i=1} + \underbrace{(2-1)^3}_{i=2} + \underbrace{(3-1)^3}_{i=3} + \cdots + \underbrace{(n-1)^3}_{i=n}$ in sigma notation.

$$\sum_{i=1}^{n} (i-1)^3$$

Riemann sums



We have a curve $f(x)$ and we want to approx the signed area of $f(x)$ on the interval $[a, b]$

signed area :

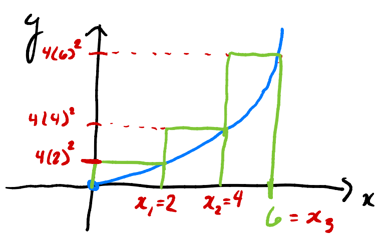
area above x-axis is positive

area below x-axis is negative

signed area $\approx$ $\sum$ of $\overset{\text{area of}}{\wedge}$ rect. above x-axis

$-\sum$ of area of rect below x-axis

(3) Approximate the signed area under $y = 4x^2$ on $[0, 6]$ use 3 rectangles.



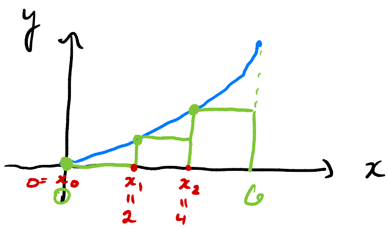
Right Riemann sum

$\Delta x :=$ width of the rectangles

$= \dfrac{6-0}{3} = 2$

$$4(2)^2 \cdot 2 + 4(4)^2 \cdot 2 + 4(6)^2 \cdot 2 = \sum_{i=1}^{3} y(\overbrace{0+i\Delta x}^{x_i})\Delta x$$

left Riemann sums



$\Delta x = 2$

$4(0)^2 \cdot 2 + 4(2)^2 \cdot 2 + 4(4)^2 \cdot 2$

$= \sum_{i=0}^{2} y(0 + i\Delta x)\Delta x$

General Case: approx area under $f(x)$ on the interval $[a, b]$ w/ n rectangles.

Right Riemann Sum:

$$\sum_{i=1}^{n} f(a + i\Delta x)\Delta x$$

$$\Delta x = \frac{b-a}{n}$$

Left Riemann Sum

$$\sum_{i=0}^{n-1} f(a + i\Delta x)\Delta x$$