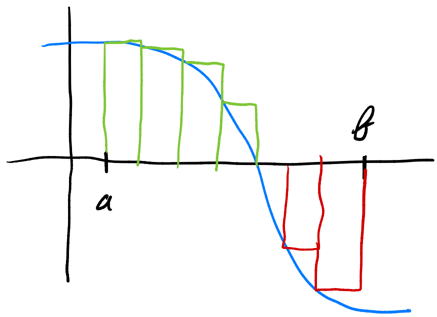
Lesson 29: Definite integration I

Last time we approximated signed areas under a curves



Approximated signed area $=$ $\sum$ rect. on top $- \sum$ rect on bot

More rect $=$ better approx.

What if we let the number of rectangles $\to \infty$

$$\text{Signed area under } f(x) = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

Def. The **definite integral** of $f(x)$ on $[a,b]$ is the signed area under the curve $f(x)$ on $[a,b]$.

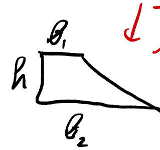
$$\int_a^b f(x)\,dx$$

a called the lower bound
b called the upper bound

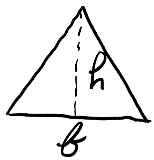
$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

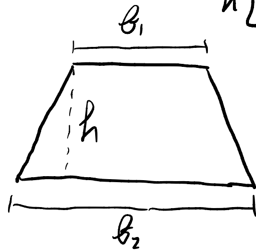
Next week we will do general case, but today we will simple geometric shapes to calculate signed area.

Still trapezoid



Recall:

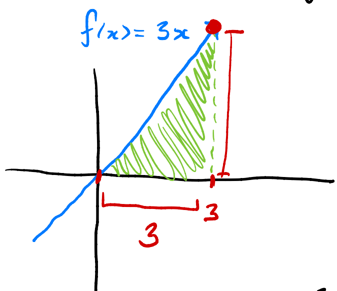




$A = lw$ $A = \frac{1}{2} bh$ $A = \frac{b_1 + b_2}{2} h$

① Evaluate $\int_0^3 3x \, dx$ using geometrical shapes.

$\int_0^3 3x \, dx$ = signed area under $3x$ on $[0, 3]$



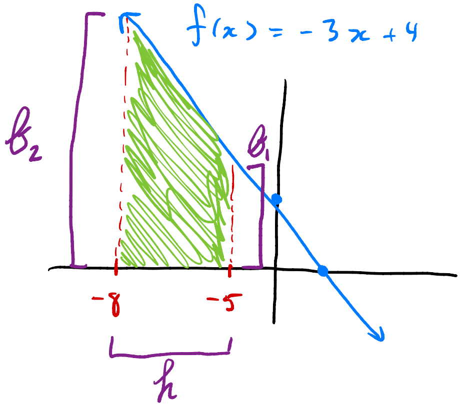
= area of the triangle

$b = 3$ $h = f(3) = 3 \cdot 3 = 9$

$$\int_0^3 3x \, dx = \frac{1}{2}(3)(9) = 27/2 \checkmark$$

(2) Evaluate $\int_{-8}^{-5} -3x+4\,dx$ using geometrical shapes.

$\int_{-8}^{-5} -3x+4\,dx$ = signed area under $f(x) = -3x+4$ on the interval $[-8,-5]$.



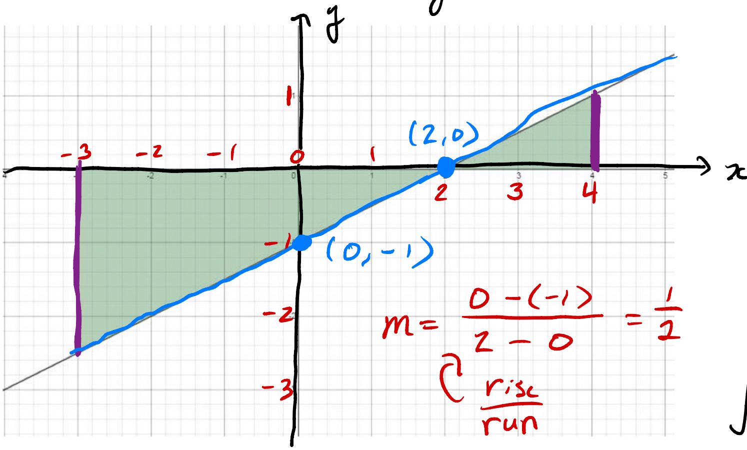
= area of our shaded region.

$h = 3$, $b_1 = f(-5) = 15+4 = 19$

$b_2 = f(-8) = 24+4 = 28$

$$\int_{-8}^{-5} -3x+4\,dx = \frac{b_1+b_2}{2}h = \frac{19+28}{2}\cdot 3 = \frac{141}{2}$$ ✓

(3) Construct the definite integral of the shaded region.



$m = \frac{0-(-1)}{2-0} = \frac{1}{2}$

$\frac{\text{rise}}{\text{run}}$

interval of integration is $[-3,4]$

Function:

$y - y_1 = m(x - x_1)$

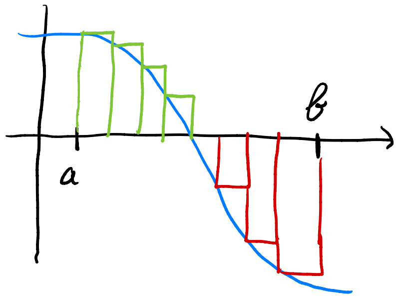
$y - 0 = m(x-2)$

$y = \frac{1}{2}(x-2)$

$$\int_{-3}^{4} \frac{1}{2}(x-2)\,dx$$

Lesson 29: Definite Integration I

Last time we approx. signed area under curves.



signed area $\approx$ $\sum$ area of rect above x-axis

$-$ $\sum$ area of rect below x-axis

More rectangles = better approx.

What happens if the number rectangles $\to \infty$?

$x_i = a + i\Delta x$

$$\text{signed area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

Def. The **definite integral** of $f(x)$ on $[a,b]$ is the signed area under the curve $f(x)$ on $[a,b]$. It is denoted by

$$\int_a^b f(x)\,dx$$

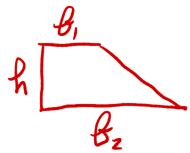
a called the lower bound

b called the upper bound

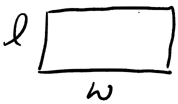
So $$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x$$

Next week we will do the general case, but today we will use high school geometry to find the signed areas.

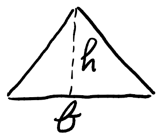
this also fine



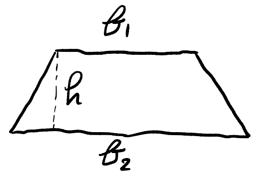
Recall:



$A = lw$

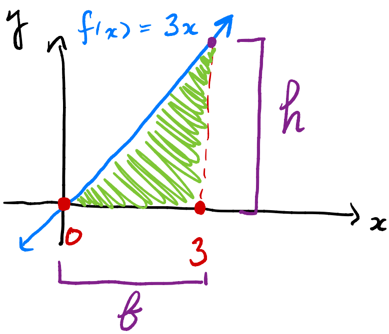


$A = \frac{1}{2} b h$



$A = \frac{b_1 + b_2}{2} h$

① Evaluate $\int_0^3 3x \, dx$ using geometrical shapes.



$b = 3 \quad h = f(3) = 3 \cdot 3 = 9$

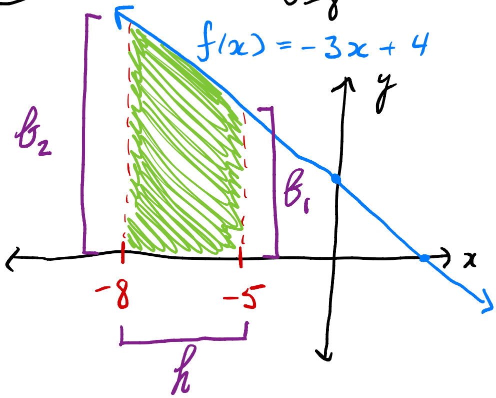
$\int_0^3 3x\,dx$ = signed area under $f(x) = 3x$ on $[0, 3]$

= area of the shaded triangle.

$= \frac{1}{2} b h$

$= \frac{1}{2}(3)(9) = 27/2$ ✓

② Evaluate $\int_{-8}^{-5} -3x + 4 \, dx$ using geometrical shapes



$$\int_{-8}^{-5} -3x + 4 dx = \text{area of the shaded trapezoid.}$$

$$= \frac{B_1 + B_2}{2} h$$

$$= \frac{19 + 28}{2} (3)$$

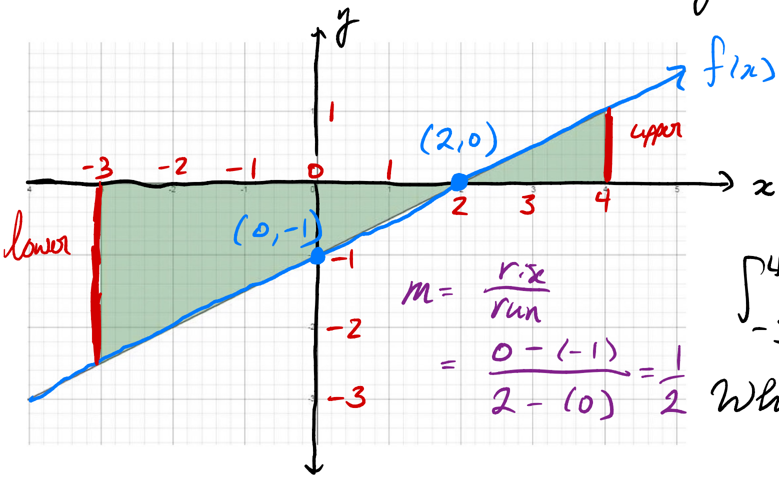
$$= \frac{141}{2}$$

$h = 3$ $\quad B_1 = f(-5) = 15 + 4 = 19$

$B_2 = f(-8) = 24 + 4 = 28$

h, B_1, B_2 are distances so they are always positive

③ Construct the definite integral of the shaded region



$$m = \frac{\text{rise}}{\text{run}} = \frac{0 - (-1)}{2 - (0)} = \frac{1}{2}$$

interval of integration is $[-3, 4]$

$$\int_{-3}^{4} f(x) \, dx$$

What is $f(x)$?

point-slope form: $y - y_1 = m(x - x_1)$

$y - 0 = m(x - 2)$

$y = \frac{1}{2}(x - 2)$

$$\int_{-3}^{4} \frac{1}{2}(x-2) \, dx \quad ✓$$