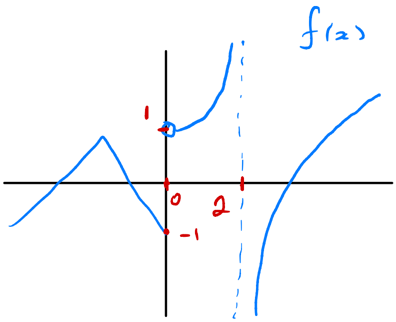
DNE vs undef.

limits ~ DNE   functions ~ undef



$$\lim_{x \to 2} f(x) = DNE$$

$$\lim_{x \to 2^-} f(x) = +\infty$$

$$\lim_{x \to 2^+} f(x) = -\infty$$

$$\lim_{x \to 0} f(x) = DNE \quad , \quad f(0) = -1$$

$$\lim_{x \to 0^-} f(x) = -1 \quad , \quad \lim_{x \to 0^+} f(x) = 1$$

Lesson #3: finding limits Analytically

$$\lim_{x \to c} f(x)$$

f is not piecewise

Case 1: $f(c)$ is a number (possibly $= 0$)

① $f(x) = 3x^2 - 1$, $\lim_{x \to 2} f(x)$

$$\lim_{x \to 2} f(x) = 3(2)^2 - 1 = 11$$

② $f(x) = \frac{1}{x^2+1}$, $\lim_{x \to -1} f(x)$ $f(-1) = \frac{1}{(-1)^2+1} = \frac{1}{2}$

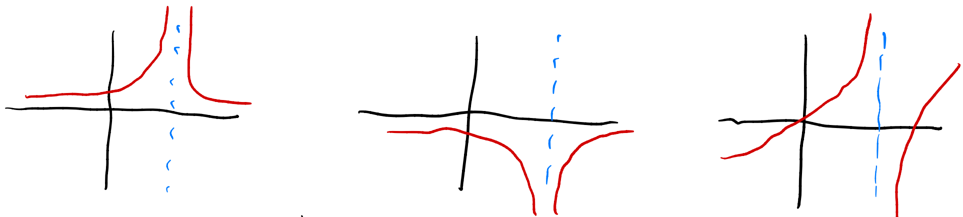
$$\lim_{x \to -1} f(x) = 1/2$$

③ $f(x) = \cos x$, $\lim_{x \to \pi/2} f(x)$

$\cos(\pi/2) = 0$, $\lim_{x \to \pi/2} f(x) = 0$

Case 2: $f(c) = \dfrac{\text{nonzero number}}{0}$

This means that f has a vertical asymptote at $x = c$, $\lim_{x \to c} f(x)$ is either $\pm\infty$ or DNE



④ $f(x) = \dfrac{1}{(x-5)^2}$, $\lim_{x \to 5} f(x)$

$f(5) = \dfrac{1}{(5-5)^2} = \dfrac{1}{0}$

x	4.9	4.99	5	5.01	5.1
$f(x)$	100	10,000		10,000	100

$\longrightarrow +\infty$ $\quad$ $+\infty \longleftarrow$

$\lim_{x \to 5} f(x) = +\infty$

$f(5) =$ undef.

Case 3: $f(c) = \frac{0}{0}$

Either f has a "hole at c" or an "asymptote at c"

(5) $f(x) = \frac{x^2 - 7x}{x^2 + 3x}$, $\lim_{x \to 0} f(x)$

$f(0) = \frac{0^2 - 7 \cdot 0}{0^2 + 3 \cdot 0} = \frac{0}{0}$

try and factor, then reduce

$f(x) = \frac{x(x-7)}{x(x+3)} = \frac{x-7}{x+3}$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{x-7}{x+3} = \frac{-7}{3}$ $\quad \frac{0-7}{0+3} = \frac{-7}{3}$

(6) $f(x) = \frac{x^2 + 5x + 6}{x^2 + 3x + 2}$, $\lim_{x \to -2} f(x)$

$f(-2) = \frac{(-2)^2 + 5(-2) + 6}{(-2)^2 + 3(-2) + 2} = \frac{4 - 10 + 6}{4 - 6 + 2} = \frac{0}{0}$

$f(x) = \frac{x^2 + 5x + 6}{x^2 + 3x + 2} = \frac{(x+2)(x+3)}{(x+2)(x+1)}$ $\quad \frac{x+3}{x+1}$

$\lim_{x \to -2} f(x) = \lim_{x \to -2} \frac{x+3}{x+1} = \frac{1}{-1} = -1$

Limits Properties

let $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = K$

* Both L, K must exist and be finite *

$$\lim_{x \to c} \left(a\, f(x) \right) = aL$$ a is a number

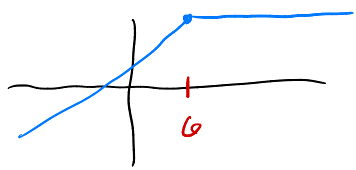
$$\lim_{x \to c} \left(f(x) + g(x) \right) = L + K$$

$$\lim_{x \to c} \left(f(x) \cdot g(x) \right) = L \cdot K$$

if $K \neq 0$, then $\lim_{x \to c} \left(\dfrac{f(x)}{g(x)} \right) = \dfrac{L}{K}$

$$\lim_{x \to c} \left[\left(f(x) \right)^n \right] = L^n$$

7) $f(x) = \begin{cases} x + 1 & x \leq 6 \\ 7 & x > 6 \end{cases}$



$\lim_{x \to 6} f(x) = 7$, $\lim_{x \to 6^-} f(x) = 6 + 1 = 7$

$\lim_{x \to 6^+} f(x) = 7$

HW2 : #9

$$f(x) = \begin{cases} 6x^2 - 3 & x \le 3 \\ 11x + 3 & x > 3 \end{cases}$$

$\lim_{x \to 3} f(x) =$ DNE

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$	47.46	50.641	50.964		36.011	36.11	37.1

$$\lim_{x \to 3^-} f(x) = 51 \neq 36 = \lim_{x \to 3^+} f(x)$$

Lesson 3: finding limits Analytically

$\lim_{x \to c} f(x)$ $f(c) = ?$

Case 1: f is not piecewise and $f(c)$ is a number ($f(c)$ may $= 0$ as well)

$$\lim_{x \to c} f(x) = f(c)$$

1. $f(x) = 3x^2 - 1$, $\lim_{x \to 2} f(x) = 11$

$f(2) = 3(2)^2 - 1 = 11$

2. $f(x) = \frac{1}{x^2 + 1}$, $\lim_{x \to -1} f(x) = \frac{1}{2}$

$f(-1) = \frac{1}{(-1)^2 + 1} = \frac{1}{2}$

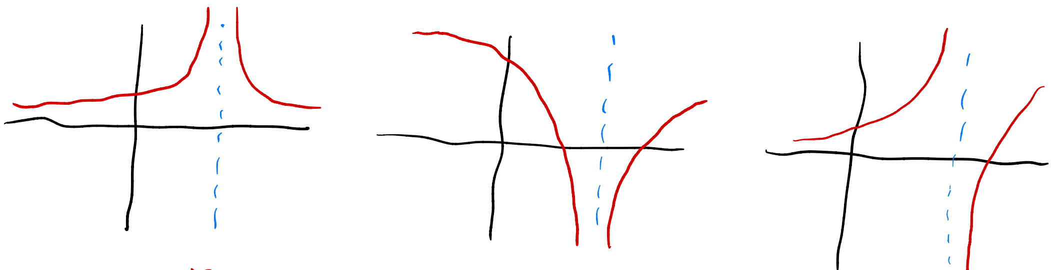
③ $f(x) = \cos x \qquad \lim_{x \to \pi/2} f(x) = 0$

$f(\pi/2) = \cos(\pi/2) = 0$

Case 2: $f(c) = \dfrac{\text{nonzero number}}{0}$

This means f has a vertical asymptote at $x = c$

$\lim_{x \to c} f(x)$ either $\pm\infty$ or DNE



$+\infty \qquad\qquad -\infty \qquad\qquad \text{DNE}$

④ $f(x) = \dfrac{1}{(x-5)^2}, \quad \lim_{x \to 5} f(x)$

$f(5) = \dfrac{1}{(5-5)^2} = \dfrac{1}{0}$

x	4.9	4.99	5	5.01	5.1
$f(x)$	100	10,000		10,000	100

$\longrightarrow +\infty \qquad +\infty \longleftarrow$

$$\lim_{x \to 5} f(x) = +\infty$$

Case 3: $f(c) = \frac{0}{0}$

this means f either has "a hole at c"
or f has an asymptote at c

first factor, then reduce

5) $f(x) = \dfrac{x^2 - 7x}{x^2 + 3x}$ $\qquad \lim_{x \to 0} f(x)$

$$f(0) = \frac{0^2 - 7 \cdot 0}{0^2 + 3 \cdot 0} = \frac{0}{0}$$

$$f(x) = \frac{x^2 - 7x}{x^2 + 3x} = \frac{x(x-7)}{x(x+3)} \qquad \frac{x-7}{x+3}$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{x-7}{x+3} = \frac{-7}{3}$$

Limit Properties

Let $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = K$

* Careful: L, K must exist and be finite *

if a is a number, then $\lim_{x \to c} \left(a f(x) \right) = aL$

$$\lim_{x \to c} \left(f(x) + g(x) \right) = L + K$$

$$\lim_{x \to c} \left(f(x) \cdot g(x) \right) = L \cdot K$$

if $k \neq 0$, then $\lim_{x \to c} \left(\frac{f(x)}{g(x)} \right) = \frac{L}{k}$

$\lim_{x \to c} \left[(f(x))^n \right] = L^n$

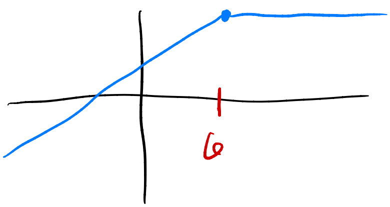
(6) $f(x) = \begin{cases} x+1 & \text{if } x \leq 6 \\ 7 & \text{if } x > 6 \end{cases}$

$\lim_{x \to 6} f(x)$

$= 7$

$\lim_{x \to 6^-} f(x) = \lim_{x \to 6^-} (x+1) = 7$

$\lim_{x \to 6^+} f(x) = \lim_{x \to 6^+} (7) = 7$



(7) $f(x) = \frac{x}{x^2}$ $\lim_{x \to 0} f(x)$

$f(0) = \frac{0}{0}$

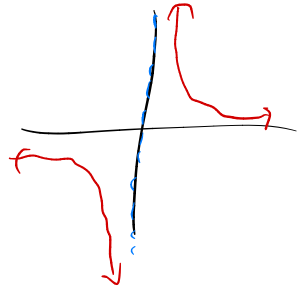
first factor, then reduce

$f(x) = \frac{x}{x \cdot x}$ $\frac{1}{x}$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{1}{x}$ $\frac{1}{0}$ Case 2!

x	-0.1	-0.01	0	$.01$	$.1$
$f(x)$	-10	-100		100	10

$-\infty$ $\quad\quad$ $+\infty$



$$\lim_{x \to 0} f(x) = \text{DNE}$$