Lesson 30: Definite Integration II

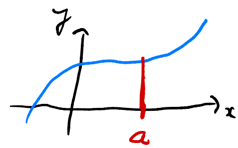
Recall: We defined the definite integral by

$$\int_a^b f(x)\,dx := \underline{\text{signed}} \text{ area under } f(x) \text{ on } [a,b]$$

Properties of the definite integral

Let a, b, c, and k be constants (numbers)

$$\int_a^a f(x)\,dx = 0$$



area of line is just zero

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

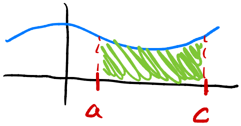
integration is orientated

$$\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$$

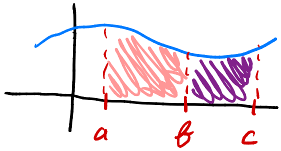
$$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

integration is a linear operator.

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$



=



Warning: $\displaystyle\int_a^b f(x)\,g(x)\,dx \neq \int_a^b f(x)\,dx \int_a^b g(x)\,dx$

① Let $\int_5^9 2x^3\,dx = 2968$. Find $\int_9^5 2x^3\,dx$

and $\int_5^9 20x^3\,dx$.

$$\int_9^5 2x^3\,dx = -\int_5^9 2x^3\,dx = -2968$$

$$\int_5^9 20x^3\,dx = \int_5^9 10\cdot 2x^3\,dx$$

$$= 10\int_5^9 2x^3\,dx$$

$$= 29680.$$

② Let $\int_1^2 x^2\,dx = 7/3$, $\int_1^2 x\,dx = 3/2$, $\int_1^2 dx = 1$

Compute $\int_1^2 -4x^2 + 3x - 3\,dx$.

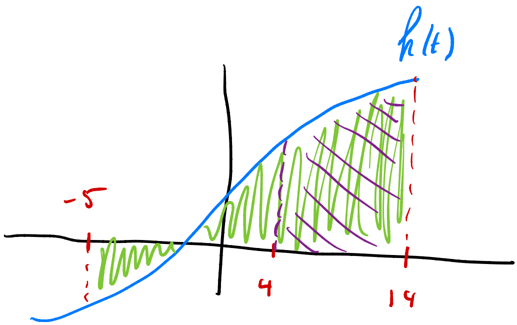
$$= \int_1^2 -4x^2\,dx + \int_1^2 3x\,dx + \int_1^2 -3\,dx$$

$$= -4\int_1^2 x^2\,dx + 3\int_1^2 x\,dx - 3\int_1^2 dx$$

$$= -4\left(7/3\right) + 3\left(3/2\right) - 3(1)$$

$$= \frac{-28}{3} + \frac{9}{2} - 3 \quad \checkmark$$

③ If $\int_{-5}^{14} h(t)\,dt = 28$, $\int_{4}^{14} h(t)\,dt = 36$, then find $\int_{-5}^{4} h(t)\,dt$.



$\int_{-5}^{14} h(t)\,dt = 28$

$\int_{4}^{14} h(t)\,dt = 36$

$$\int_{-5}^{4} h(t)\,dt = \int_{-5}^{14} h(t)\,dt - \int_{4}^{14} h(t)\,dt$$

$$= 28 - 36$$

$$= -8$$

④ If $\int_{5}^{7} f(x)\,dx = 1$, $\int_{7}^{12} f(x)\,dx = 5$, $\int_{5}^{12} g(x)\,dx = 14$ then find $\int_{5}^{12} 7f(x) - 6g(x)\,dx$.

$$\int_{5}^{12} f(x)\,dx = \int_{5}^{7} f(x)\,dx + \int_{7}^{12} f(x)\,dx = 6$$

$$\int_{5}^{12} 7f(x) - 6g(x)\,dx = 7\overbrace{\int_{5}^{12} f(x)\,dx}^{6} - 6\overbrace{\int_{5}^{12} g(x)\,dx}^{14} = -42$$

(5) $\int_a^b g(x)\,dx = 2$, $\int_a^c g(x)\,dx = 3\int_a^b g(x)\,dx$

Compute $\int_b^c g(x)\,dx$

$$2 = \int_a^b g(x)\,dx = \int_a^c g(x)\,dx + \int_c^b g(x)\,dx$$

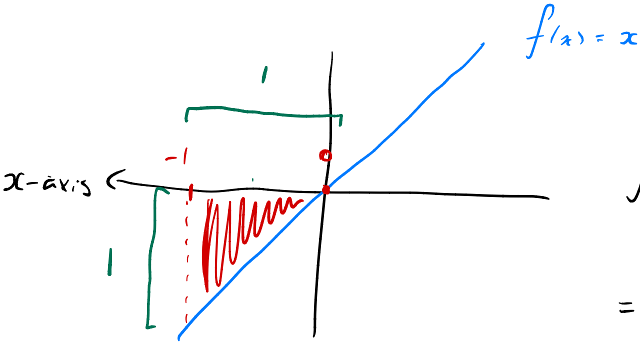
$$2 = \int_a^c g(x)\,dx - \int_b^c g(x)\,dx$$

$$\int_b^c g(x)\,dx = \int_a^c g(x)\,dx - 2$$

$$= 3\int_a^b g(x)\,dx - 2$$

$$= 3(2) - 2$$

$$= 4$$



$$\int_{-1}^{0} f(x)\,dx$$

$= -($area of the shaded region$)$

$$= -\frac{1}{2}(1)(1) = -\frac{1}{2}$$

Lesson 30: Definite Integration II

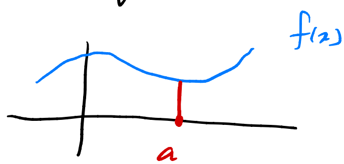
Recall: We define the definite integral by

$$\int_a^b f(x)\,dx := \underline{\underline{\text{signed}}} \text{ area under } f(x) \text{ on } [a,b]$$

Properties of the definite integral

a, b, c, k numbers

$$\int_a^a f(x)\,dx = 0$$



lines have zero area.

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

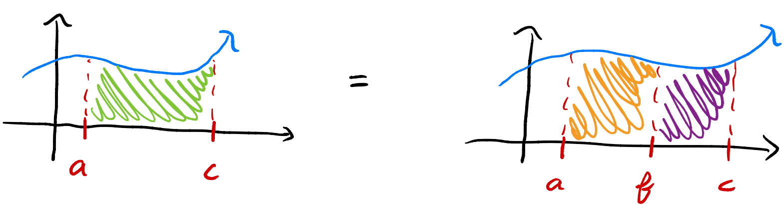
integration is orientated.

$$\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

integration is a linear operator.

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$



Warning: $\int_a^b f(x)g(x)\,dx \neq \int_a^b f(x)\,dx \int_a^b g(x)\,dx$

① Let $\int_5^9 2x^3\,dx = 2968$,

Compute $\int_9^5 2x^3\,dx$ and $\int_5^9 20x^3\,dx$.

$$\int_9^5 2x^3\,dx = -\int_5^9 2x^3\,dx = -2968$$

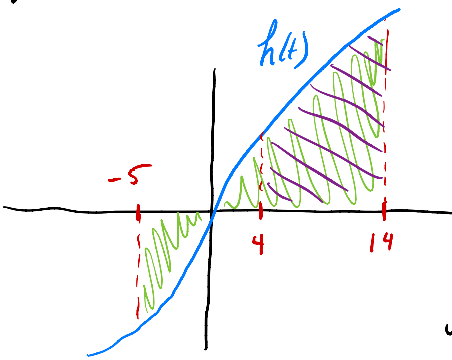
$$\int_5^9 20x^3\,dx = \int_5^9 10\cdot 2x^3\,dx$$
$$= 10\int_5^9 2x^3\,dx$$
$$= 29680$$

② Let $\int_1^2 x^2\,dx = 7/3$, $\int_1^2 x\,dx = 3/2$, $\int_1^2 dx = 1$

Compute $\int_1^2 -4x^2 + 3x - 3\,dx$

$$= \int_1^2 -4x^2\,dx + \int_1^2 3x\,dx + \int_1^2 -3\,dx$$
$$= -4\int_1^2 x^2\,dx + 3\int_1^2 x\,dx - 3\int_1^2 dx$$
$$= -4(7/3) + 3(3/2) - 3(1)$$
$$= -\frac{28}{3} + \frac{9}{2} - 3$$

③ If $\int_{-5}^{14} h(t)\,dt = 28$, $\int_{4}^{14} h(t)\,dt = 36$, then find $\int_{-5}^{4} h(t)\,dt$.



$$\int_{-5}^{14} h(t)\,dt = 28$$

$$\int_{4}^{14} h(t)\,dt = 36$$

$$\int_{-5}^{4} h(t)\,dt = \int_{-5}^{14} h(t)\,dt - \int_{4}^{14} h(t)\,dt$$

$$= 28 - 36$$

$$= -8$$

④ If $\int_{5}^{7} f(x)\,dx = 1$, $\int_{7}^{12} f(x)\,dx = 5$, $\int_{5}^{12} g(x)\,dx = 14$ then compute $\int_{5}^{12} 7f(x) - 6g(x)\,dx$

$$\int_{5}^{12} f(x)\,dx = \int_{5}^{7} f(x)\,dx + \int_{7}^{12} f(x)\,dx$$

$$= 1 + 5 = 6$$

$$\int_{5}^{12} 7f(x) - 6g(x)\,dx = 7\int_{5}^{12} f(x)\,dx - 6\int_{5}^{12} g(x)\,dx$$

$= 7(6) - 6(14)$

$= -42$

5. Let $\int_a^b g(x)\,dx = 2$, $\int_a^c g(x)\,dx = 3\int_a^b g(x)\,dx$

find $\int_b^c g(x)\,dx$.

$\int_a^c g(x)\,dx = 3(2) = 6$

$6 = \int_a^c g(x)\,dx = \int_a^b g(x)\,dx + \int_b^c g(x)\,dx$

$6 = 2 + \int_b^c g(x)\,dx$

$\int_b^c g(x)\,dx = 4$