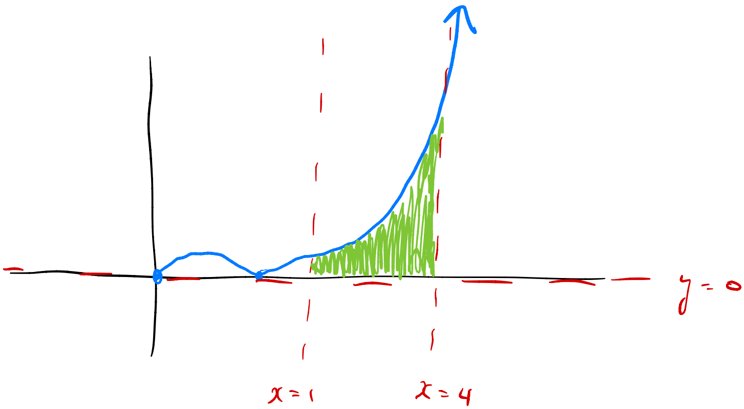
HW 31 #10 Find area enclosed by the following

$y = 6\left(\frac{\sqrt{x}}{5} - \frac{x}{3}\right)^2, \quad y = 0, \quad x = 1, \quad x = 4$



$$\int_1^4 6\left(\frac{\sqrt{x}}{5} - \frac{x}{3}\right)^2 dx = 6\int_1^4 \frac{x}{25} - \frac{2x^{3/2}}{15} + \frac{x^2}{9}\,dx$$

Lesson 32: Fundamental Theorem of Calculus II

Recall: Let $f(x)$ be a function with antiderivative $F(x)$, then

(FTC) $$\int_a^b f(x)\,dx = F(x)\Big|_a^b := F(b) - F(a)$$

(1) The growth rate of the pop. of a country is $P(t) = \sqrt{t}\,(2370t + 6270)$ where t is time in years.

How much does the pop. increase from year $t=1$ to year $t=4$?

Integral from a to b = net change from a to b.

$$\int_1^4 \sqrt{t}\,(2370t + 6270)\,dt \qquad \sqrt{t} = t^{1/2}$$

$n \neq -1$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$= \int_1^4 2370t^{3/2} + 6270t^{1/2}\,dt$$

$$= \left(2370 \cdot \frac{2}{5} \cdot t^{5/2} + 6270 \cdot \frac{2}{3} \cdot t^{3/2}\right)\Big|_1^4$$

$$\left(2370 \cdot \frac{2}{5}(4)^{5/2} + 6270 \cdot \frac{2}{3} \cdot (4)^{3/2}\right)$$

$$- \left(2370 \cdot \frac{2}{5} \cdot (1)^{5/2} + 6270 \cdot \frac{2}{3}(1)^{3/2}\right)$$

$$= 2370 \cdot \frac{2^6}{5} + 6270 \cdot \frac{2^4}{3} - 2370 \cdot \frac{2}{5} - 6270 \cdot \frac{2}{3}$$

$$= 58648$$

(2) The velocity function in meters/min, of a particle moving in a straight line is $v(t) = 5t - 3$, where t time in minutes.

a) Find the $\underbrace{\text{displacement}}_{}$ of the particle from $t=2$ to $t=5$ minutes.

net change (of position)

$$\text{displacement} = \int_2^5 5t - 3 \, dt = \underline{43.5 \text{ meters}}$$

b) Find the time x when the displacement is zero after the particle starts moving.

displacement function $= \int_0^x 5t-3\,dt = 0$

$$\int_0^x 5t-3\,dt = \left(\frac{5}{2}t^2 - 3t\right)\Big|_0^x$$
$$= \left(\frac{5}{2}x^2 - 3x\right) - (0-0)$$
$$= \frac{5}{2}x^2 - 3x$$

$$\frac{5}{2}x^2 - 3x = 0$$

$$\frac{5}{2}x\left(x - \frac{6}{5}\right) = 0$$

$x = 0$ minutes $\qquad \boxed{x = 6/5 = 1.2 \text{ minutes}}$

silly answer

(3) The acceleration of a car t seconds after the breaks are applied is

$a(t) = -(t-5)^2$ mph per second.

What is the decrease in velocity in mph 4 seconds after breaks are applied?

net change

$$\int_0^4 -(t-5)^2\,dt = -\int_0^4 t^2 - 10t + 25\,dt$$

$$= -\left(\frac{1}{3}t^3 - \frac{10}{2}t^2 + 25t\right)\Big|_0^4$$

$$= -\left[\left(\frac{1}{3}(4)^3 - \frac{10}{2}(4)^2 + 25(4)\right) - 0\right]$$

$$= -\left(\frac{4^3}{3} - 5\cdot 4^2 + 25\cdot 4\right)$$

$$= -\frac{124}{3} \text{ mph.} \leftarrow \text{net change in velocity.}$$

Decrease in velocity is $\frac{124}{3}$ mph.

4 A faucet is turned on at 9:00 am and water flows into a tank at

$$r(t) = 7\sqrt{t} \ \ ft^3/hr.$$

t is hours after 9am.

a) How much water flows into the tank (net change) between 10 am ($\rightsquigarrow t=1$) and 1pm ($\rightsquigarrow t=4$)?

$$\int_1^4 7\sqrt{t}\,dt = 32.\overline{6} \ \ ft^3.$$

Lesson 32: Fundamental Theorem of Calculus II

Recall: Let $f(x)$ be a function with an antiderivative $F(x)$, then

(FTC) $$\int_a^b f(x)\,dx = F(x)\Big|_a^b := F(b) - F(a)$$

Main Idea: integral from a to b
= net change from a to b

(1) The growth rate of a pop. of a country is $P(t) = \sqrt{t}\,(2370t + 6270)$ where t is time in years.

How much does the pop. increase (net change) from year $t=1$ to year $t=4$?

$\sqrt{t} = t^{1/2}$

net change of pop from $t=1 \to t=4$ $$= \int_1^4 \sqrt{t}\,(2370t + 6270)\,dt$$

$$= \int_1^4 2370t^{3/2} + 6270t^{1/2}\,dt$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$= \left(2370 \cdot \frac{2}{5} \cdot t^{5/2} + 6270 \cdot \frac{2}{3} \cdot t^{3/2}\right)\Bigg|_1^4$$

$$= \left(2370 \cdot \frac{2}{5}(4)^{5/2} + 6270 \cdot \frac{2}{3} \cdot (4)^{3/2}\right)$$

$$- \left(2370 \cdot \frac{2}{5}(1)^{5/2} + 6270 \cdot \frac{2}{3} \cdot (1)^{3/2}\right)$$

$$= 2370 \cdot \frac{2^6}{5} + 6270 \cdot \frac{2^4}{3} - 2370 \cdot \frac{2}{5} - 6270 \cdot \frac{2}{3}$$

$= 58648$ people. ✓

(2) The velocity function, in meters/minute, of a particle moving in a straight line is $v(t) = 5t - 3$, where t is in minutes.

a) Find the displacement of the particle from $t=2$ to $t=5$ minutes.

net change

displacement from $t=2$ to $t=5$ $= \displaystyle\int_2^5 5t - 3 \, dt = 43.5$ meters.

b) Find the time x when the displacement is zero after the particle starts moving.

$$\text{displacement} = \int_0^x 5t - 3\,dt = 0$$

$$\int_0^x 5t - 3\,dt = \left(\frac{5}{2}t^2 - 3t\right)\Big|_0^x$$

$$= \frac{5}{2}x^2 - 3x - (0 - 0)$$

$$= \frac{5}{2}x^2 - 3x$$

$$\frac{5}{2}x^2 - 3x = 0$$

$$\frac{5}{2}x\left(x - \frac{6}{5}\right) = 0$$

$x = 0$ minutes or $\boxed{x = 6/5 = 1.2 \text{ minutes.}}$

↑ silly answer

③ The acceleration of a car t seconds after the breaks are applied is

$$a(t) = -(t-5)^2 \text{ mph per second.}$$

What is the decrease in velocity of the car in mph 4 seconds after the breaks are applied?

net change

net change of velocity 4 sec after breaks $= \displaystyle\int_0^4 -(t-5)^2 \, dt$

$$= -\int_0^4 t^2 - 10t + 25 \, dt$$

$$= -\left(\frac{t^3}{3} - 5t^2 + 25t\right)\Big|_0^4$$

$$= -\left[\left(\frac{4^3}{3} - 5\cdot 4^2 + 25\cdot 4\right) - (0)\right]$$

$$= -\frac{124}{3} \leftarrow \text{net change in velocity}$$

So the velocity decreases by $\frac{124}{3}$ mph.

④ A faucet is turned on at 9am and water flows into a tank at a rate of

$$r(t) = 7\sqrt{t} \ ft^3/hr$$

where t is hours after 9 am.

a) How much water flows into the tank between 10 am and 1pm?

net change

$\rightsquigarrow t=1$ $\rightsquigarrow t=4$

$$\int_1^4 7\sqrt{t}\, dt = \int_1^4 7t^{1/2}\, dt$$

$$= 7 \cdot \frac{2}{3} \cdot t^{3/2} \Big|_1^4$$

$$= \frac{14}{3}\left(4^{3/2} - 1^{3/2}\right)$$

$$= \frac{14}{3}\left(2^3 - 1\right)$$

$$= \frac{98}{3} \ ft^3$$