HW 3 #10

$$f(x) = \begin{cases} -3x - \pi/2 & x \le -\pi/2 \\ \cos x & -\pi/2 < x < \pi/2 \\ 4\sin x + 2 & x \ge \pi/2 \end{cases}$$

$\lim_{x \to -\pi/2} f(x) =$ DNE

$$\lim_{x \to -\frac{\pi}{2}^-} f(x) = \lim_{x \to -\frac{\pi}{2}^-} \left(-3x - \frac{\pi}{2}\right)$$

Case 1 $= -3\left(-\frac{\pi}{2}\right) - \frac{\pi}{2} = \pi$ ✳

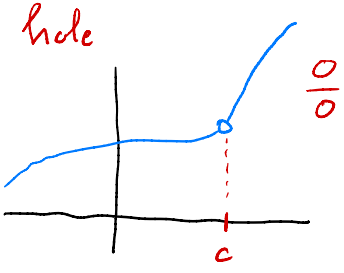
$$\lim_{x \to -\frac{\pi}{2}^+} f(x) = \lim_{x \to -\frac{\pi}{2}^+} \cos x = \cos\left(-\frac{\pi}{2}\right) = 0$$

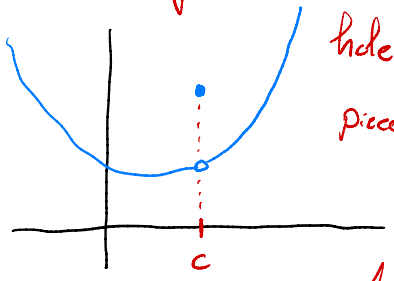
Case 1

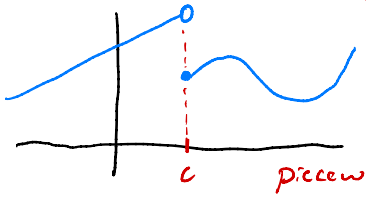
Lesson 4: Continuity

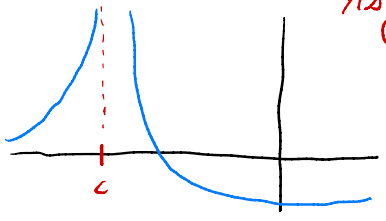
A function $f(x)$ is said to be **continuous** at $x = c$ if there are "**no breaks**" in the function

discontinuity









When is f continuous?

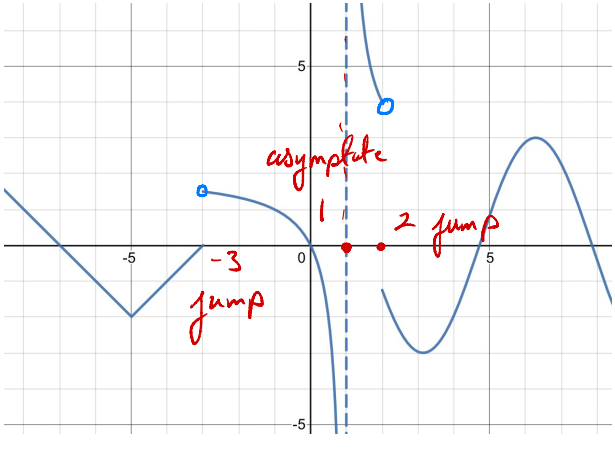
$f(x)$ is continuous at $x = c$ if

1) $f(c)$ is defined
2) $\lim_{x \to c} f(x)$ exists
3) $\lim_{x \to c} f(x) = f(c)$

} need all of these

Examples

(1)



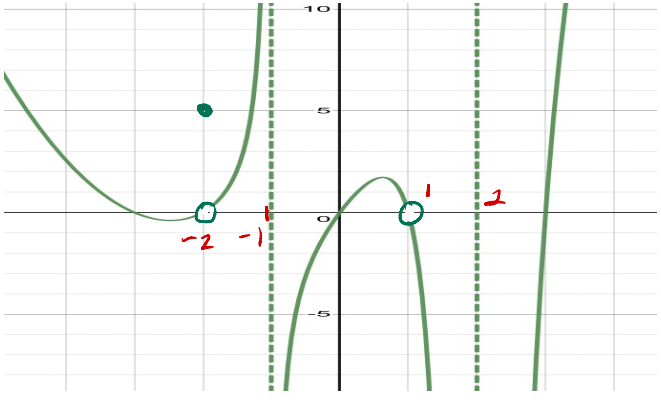
What are the x-values such that $f(x)$ is discontinuous?

at $-3, 1, 2$

$-3, 2$ are jumps

1 is asymptote

(2)



find and classify discont.

-1 : asymptote
-2 : hole
1 : hole
2 : asymptote

(3) $f(x) = \frac{1}{x-1}$, find / classify all discont.

find when $f(x) = \frac{1}{0}$

look at the denominator: $x - 1 = 0$ $\quad x = 1$

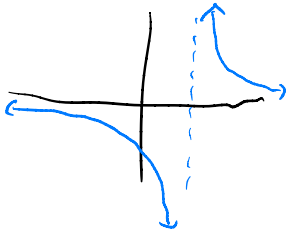
$$f(1) = \frac{1}{1-1} = \frac{1}{0}$$

f has discont at $x = 1$: asymptote

x	.9	.99	1	1.01	1.1
$f(x)$					

$\longrightarrow$ $\quad$ $\longleftarrow$

$-\infty$ $\quad$ ∞



(4) $f(x) = \frac{x^2 + 2x - 3}{x^2 + 5x - 6}$ find / classify disc.

$$= \frac{(x+3)(x-1)}{(x+6)(x-1)}$$

$\frac{x+3}{x+6}$

denom $= 0$ when $x = -6$ or 1

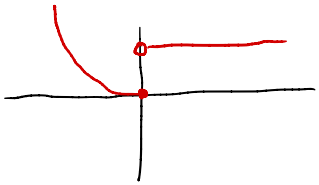
$$f(-6) = \frac{(-3)(-7)}{0} = \frac{21}{0} \Rightarrow \text{asymptote}$$

$$\frac{(1)+3}{(1)+6} = \frac{4}{7} \Rightarrow 1 \text{ is a hole}$$

(5) $f(x) = \begin{cases} x^2 & x \le 0 \\ 1 & x > 0 \end{cases}$

Check each function ✓

Check the end pts. $x = 0$



great!

$\lim_{x \to 0^-} f(x) = 0$ ✳

$\lim_{x \to 0^+} f(x) = 1$

$\lim_{x \to 0} f(x) = \text{DNE}$

f is not cont. at $x = 0$ (jump).

(6) $f(x) = \begin{cases} 2x^2 - 1 & x < 3 \\ 6x - 1 & x \ge 3 \end{cases}$

Check each "piece" ✓

Check the end pts. $x = 3$?

1) $f(3) = 6(3) - 1 = 18 - 1 = 17$ ✓

2) $\lim_{x \to 3} f(x) = 17$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (2x^2 - 1) \overset{\text{Case 1}}{=} 2(3)^2 - 1 = 17$ ✓

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (6x - 1) \overset{\text{Case 1}}{=} 17$$

3) $f(3) \overset{?}{=} \lim_{x \to 3} f(x)$ ✓

$\quad \| \qquad\qquad \|$

$\quad 17 \qquad\qquad 17$

So f is continuous at $x = 3$

⑦ $$f(x) = \begin{cases} \cos x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Is $x = 0$ discont?

1) $f(0) = 0$ ✓

2) $\lim_{x \to 0} f(x) = 1$

Case 1

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \cos x = 1$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \cos x = 1$$

3) $f(0) \neq \lim_{x \to 0} f(x)$

HW 3 #10

$$f(x) = \begin{cases} -3x - \pi/2 & x \le -\pi/2 \\ \cos x & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 4\sin x + 2 & x \ge \pi/2 \end{cases}$$

$\pi = \rho$; LON-CAPA

$$\lim_{x \to -\frac{\pi}{2}} f(x) = \text{DNE}$$

$$\lim_{x \to -\frac{\pi}{2}^-} f(x) = \lim_{x \to -\frac{\pi}{2}^-} \left(-3x - \frac{\pi}{2}\right)$$

$$\text{Case 1} = -3\left(-\frac{\pi}{2}\right) - \frac{\pi}{2}$$

$$= \pi \quad \text{not} =$$

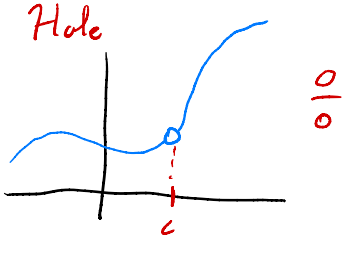
$$\lim_{x \to -\frac{\pi}{2}^+} f(x) = \lim_{x \to -\frac{\pi}{2}^+} \cos x = \cos\left(-\frac{\pi}{2}\right) = 0$$

Lesson 4: Continuity

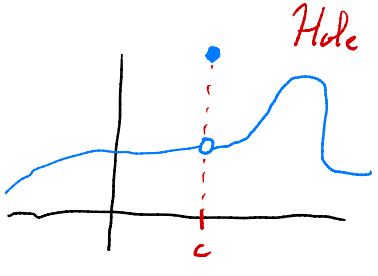
A function $f(x)$ is said to be **continuous** at $x = c$ if there are **"no Breaks"** at c

discontinuity = Break

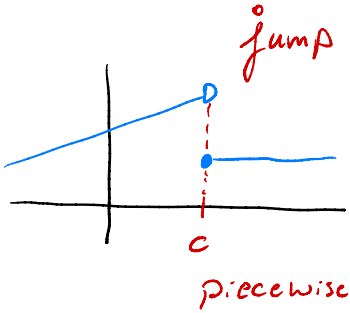
What are "Breaks"?



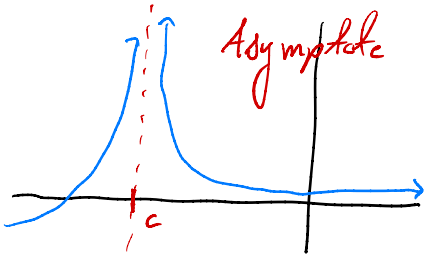
$\frac{0}{0}$



Piecewise



piecewise



$\frac{1}{0}$ or $\frac{0}{0}$

When is $f(x)$ continuous?

$f(x)$ is continuous at $x = c$ if

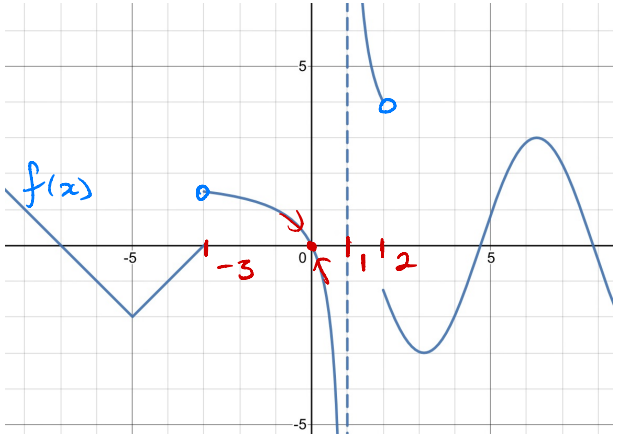
1) $f(c)$ is defined

2) $\lim_{x \to c} f(x)$ exists

3) $f(c) = \lim_{x \to c} f(x)$

need all of these to be true

Examples

① 

find / classify all discontinuities of f.

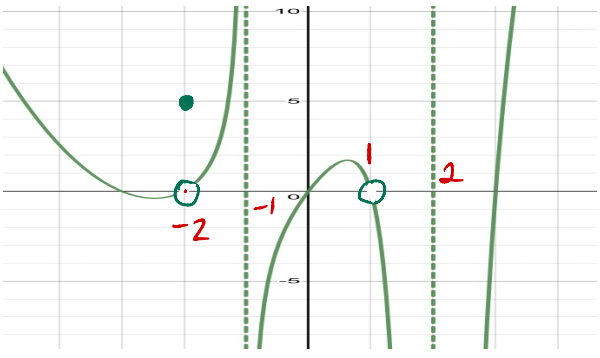
-3 : jump

1 : asymptote

2 : jump

1) $f(0) = 0$, 2) $\lim_{x \to 0} f(x) = 0$, 3) $f(0) = \lim_{x \to 0} f(x)$ ✓

(2)



find / classify discont.

-2 : hole

-1 : asymptote

1 : hole

2 : asymptote

(3) $f(x) = \dfrac{x^2 + 2x - 3}{x^2 + 5x - 6}$ find / classify discont.

$= \dfrac{(x+3)(x-1)}{(x+6)(x-1)}$

discont.

we will dividing by zero at $x = -6, 1$

$f(-6) = \dfrac{(-3)(-7)}{0} = \dfrac{21}{0}$ $x = -6$ is a asymptote

$f(1) = \dfrac{0}{0}$?

$\lim\limits_{x \to 1} f(x) = \lim\limits_{x \to 1} \dfrac{x+3}{x+6} = \dfrac{4}{7}$

So we have a hole at $x = 1$

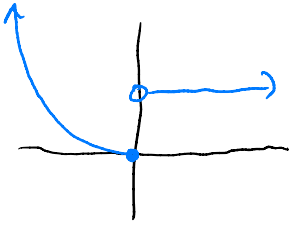
(4) $f(x) = \begin{cases} x^2 & x \le 0 \\ 1 & x > 0 \end{cases}$

find / classify discont.

Check each piece for discont

Check each end pt for discont. $x = 0$

Is f cont. at $x = 0$?



1) $f(0) = 0$

2) $\lim_{x \to 0} f(x) =$ DNE! ✗

Case 1

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2 = 0$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 1 = 1$$

Since left and right limits are numbers and are not equal, then $x = 0$ is a jump

(5) $f(x) = \begin{cases} \cos x & x \neq 0 \\ 0 & x = 0 \end{cases}$ find / classify discont.

Check pieces ✓

Check end pts $x = 0$

1) $f(0) = 0$ ✓

2) $\lim_{x \to 0} f(x) = 1$ ✓

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \cos x = \cos(0) = 1$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \cos x = \cos(0) = 1$ ✓

3) $f(0) \overset{?}{\neq} \lim_{x \to 0} f(x)$ ✗

$f(0) = 0$, $\lim_{x \to 0} f(x) = 1$

f has a discont at $x = 0$ and it is a hole

$f(x) = \begin{cases} \cos x & x \neq 0 \\ 0 & x = 0 \end{cases}$