HW 4 #6

$$f(x) = \frac{x^2+2x-3}{x^2+5x-6}$$

$$f(x) = \frac{(x+3)(x-1)}{(x+6)(x-1)}$$

-6 $\quad$ 1

$$f(-6) = \frac{(-3)\cdot(-7)}{0}$$

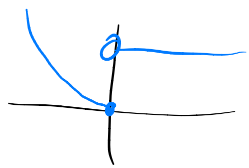
$$f(1) = \frac{0}{0}$$

$x=-6$: asymptote

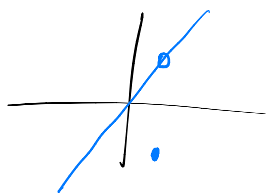
$x=1$: $\displaystyle\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x+3}{x+6} = \frac{4}{7}$

Since lim exists (not $\pm\infty$), then we have hole at $x=1$

$$g(x) = \begin{cases} x^2 & x \le 0 \\ 1 & x > 0 \end{cases}$$



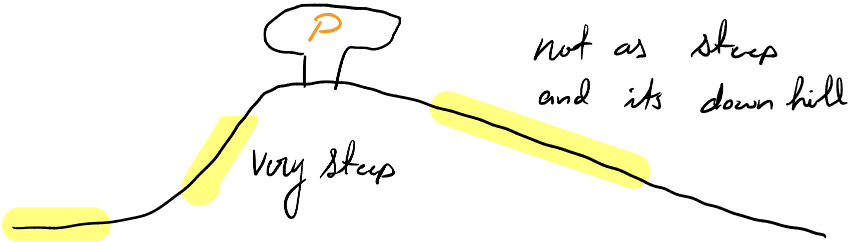
$$h(x) = \begin{cases} 2x & x \ne 1 \\ -3 & x = 1 \end{cases}$$



Lesson 5: The derivative

How steep is the hill?

Tower Drive



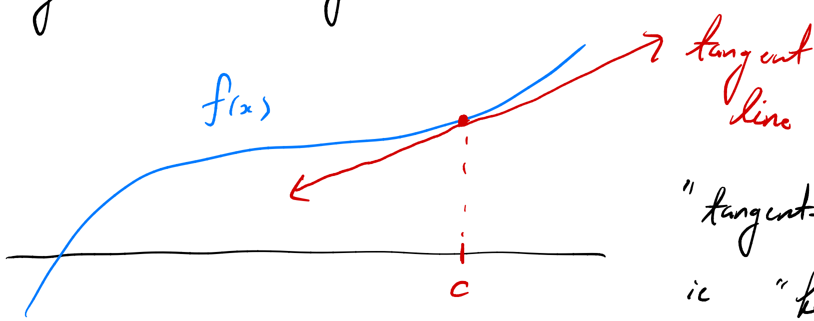
flat = 0 steepness

Derivative of $f(x)$ at c measures how steep the function is at $x = c$.

Call the steepness $f'(c)$

$$\left(\frac{d}{dx}\left[f(x)\right]\Big|_{x=c} \right)$$

Tangent into tangent lines



"tangential" to f

ie "kisses" f

$f'(c)$ = slope of tangent line

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ definition of derivative

① $f(x) = 1 + 5x$ find slope of the tangent line at $x = 0$

$$f(x+h) = 1 + 5(x+h)$$

$$f(x) = 1 + 5x$$

slope of tangent line at $x = 0$ $= \lim_{h \to 0} \dfrac{1 + 5(x+h) - (1+5x)}{h}$

$$= \lim_{h \to 0} \frac{\cancel{1} + \cancel{5x} + 5h \cancel{-1} - \cancel{5x}}{h}$$

$$= \lim_{h \to 0} \frac{5h}{h}$$

$$= \lim_{h \to 0} 5 \quad = 5$$

② $f(x) = \dfrac{3}{6x - 5}$ find the derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = \frac{3}{6(x+h) - 5}$$

$$f(x+h) - f(x) = \frac{3}{6(x+h) - 5} - \frac{3}{6x - 5}$$

$$= \frac{3}{6x + 6h - 5} - \frac{3}{6x - 5}$$

$$= \frac{3(6x-5) - 3(6x+6h-5)}{(6x+6h-5)(6x-5)}$$

$$1$$

$$f'(x) = \lim_{h\to 0} \frac{\frac{3(6x-5) - 3(6x+6h-5)}{(6x+6h-5)(6x-5)}}{h} \qquad \cdot \frac{\frac{1}{h}}{\frac{1}{h}}$$

$$= \lim_{h\to 0} \frac{3(6x-5) - 3(6x+6h-5)}{h(6x+6h-5)(6x-5)}$$

$$= \lim_{h\to 0} \frac{18x - 15 - 18x - 18h + 15}{h(6x+6h-5)(6x-5)}$$

$$= \lim_{h\to 0} \frac{-18h}{h(6x+6h-5)(6x-5)}$$

$$= \lim_{h\to 0} \frac{-18}{(6x+6h-5)(6x-5)}$$

$$= \frac{-18}{(6x+6\cdot 0-5)(6x-5)}$$

$$= \frac{-18}{(6x-5)^2}$$

③ find the tangent line of $f(x) = -2x^2 - 8$ at $x = -8$

slope of tan line at $x = -8$ $= f'(-8)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{-2(x+h)^2 - 8 - (-2x^2 - 8)}{h}$$

$$= \lim_{h \to 0} \frac{-2(x^2 + 2xh + h^2) - 8 + 2x^2 + 8}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{-2x^2} - 4xh - 2h^2 \cancel{- 8} + \cancel{2x^2} \cancel{+ 8}}{h}$$

$$= \lim_{h \to 0} \frac{-4xh - 2h^2}{h}$$

$$= \lim_{h \to 0} \left(-4x - 2h \right)$$

$$= -4x$$

$f'(x) = -4x$

$f'(-8) = 32$

Point - Slope form

- m slope
- (x_1, y_1) a pt

then $y - y_1 = m(x - x_1)$

$(-8, \underline{f(-8)})$

$f(x) = -2x^2 - 8$

$f(-8) = -2(-8)^2 - 8$

$= -128 - 8$

$= -136$

$y - (-136) = 32(x - (-8))$

$y = 32(x + 8) - 136$

tangent line !

HW 4 # 11

$$f(x) = \begin{cases} -8x - \pi/2 & x \le -\pi/2 \\ \cos x & -\frac{\pi}{2} < x < \pi/2 \\ 9\sin x + 3 & x \ge \pi/2 \end{cases}$$

find / classify discontinuities

think "piecewise" or "dividing by 0"

possible discont at $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$

Recall f cont at $x = c$ if

1) $f(c)$ is defined

2) $\lim_{x \to c} f(x)$ exists

3) $f(c) = \lim_{x \to c} f(x)$

$x = -\frac{\pi}{2}$

1) $f(-\frac{\pi}{2}) = \frac{7\pi}{2}$

2) $\lim_{x \to -\frac{\pi}{2}^-} f(x) = \frac{7\pi}{2}$

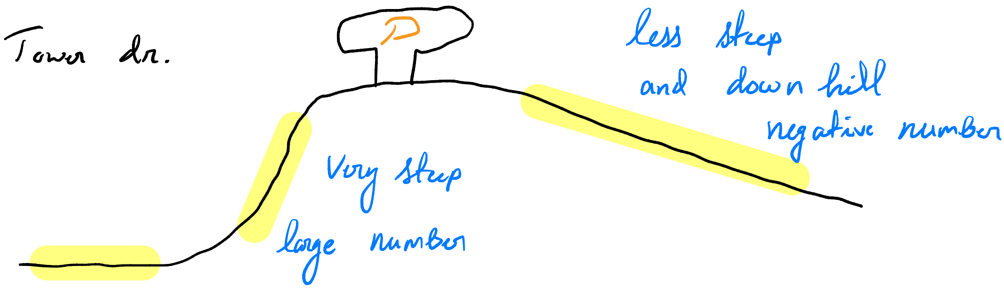
$\lim_{x \to -\frac{\pi}{2}^+} f(x) = \lim_{x \to -\frac{\pi}{2}^+} \cos x = 0$

So $\lim_{x \to -\frac{\pi}{2}} f(x)$ DNE

So $x = -\frac{\pi}{2}$ is a discont. (jump).

Lesson 5 : The derivative

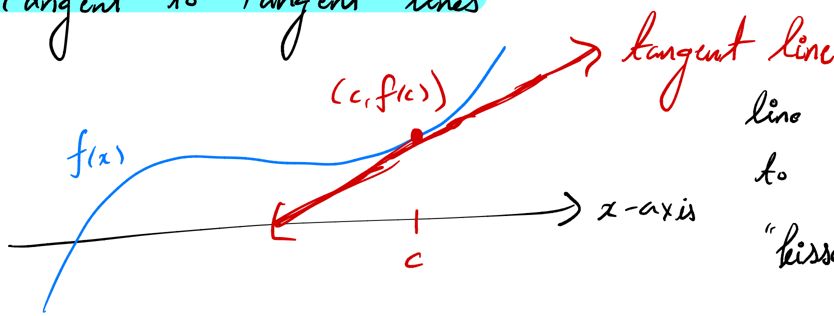
How steep is the hill?



flat steepness = 0

Derivative of $f(x)$ at $x = c$ is the steepness of f at pt. $x = c$. $f'(c)$ or $\left.\frac{d}{dx}[f(x)]\right|_{x=c}$

Tangent to Tangent lines



line tangential to $(c, f(c))$ or "kisses" $f(x)$ at $x = c$

$$f'(c) := \text{slope of the tangent line}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ definition of the derivative

① $f(x) = 1 + 5x$ find the slope of the tangent line at $x = 0$.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{1 + 5(x+h) - (1+5x)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{1} + \cancel{5x} + 5h - \cancel{1} - \cancel{5x}}{h}$$

$$= \lim_{h\to 0} \frac{5\cancel{h}}{\cancel{h}} = \lim_{h\to 0} 5 = 5$$

$f'(x) = 5$ $\qquad$ $f'(0) = 5$

Slope of tan. line at $x=0$ is $f'(0) = 5$.

② $f(x) = \dfrac{3}{6x-5}$, find $f'(x)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{3}{6(x+h)-5} - \dfrac{3}{6x-5}}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{3(6x-5) - 3(6(x+h)-5)}{(6(x+h)-5)(6x-5)}}{h}$$

$$\frac{\frac{1}{h}}{\frac{1}{h}}$$

$$= \lim_{h \to 0} \frac{3(6x-5) - 3(6(x+h)-5)}{h(6(x+h)-5)(6x-5)}$$

$$= \lim_{h \to 0} \frac{\cancel{18x} - \cancel{15} - \cancel{18x} - 18h + \cancel{15}}{h(6x+6h-5)(6x-5)}$$

$$= \lim_{h \to 0} \frac{-18h}{h(6x+6h-5)(6x-5)}$$

$$= \lim_{h \to 0} \frac{-18}{(6x+6h-5)(6x-5)}$$

$$= \frac{-18}{(6x+6 \cdot 0-5)(6x-5)} = \frac{-18}{(6x-5)^2}$$

③ find the tangent line to $f(x) = -2x^2 - 8$ at $x = 0$.

Point - Slope formula

- m slope
- (x_1, y_1)

$$y - y_1 = m(x - x_1)$$

$$(0, f(0)) = (0, -8)$$

$$m = f'(0)$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{-2(x+h)^2 - 8 - (-2x^2 - 8)}{h}$$

$$= \lim_{h \to 0} \frac{-2(x^2 + 2xh + h^2) - 8 + 2x^2 + 8}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{-2x^2} - 4xh - 2h^2 \cancel{-8} + \cancel{2x^2} + \cancel{8}}{h}$$

$$= \lim_{h \to 0} \frac{-4xh - 2h^2}{h}$$

$$= \lim_{h \to 0} \left(-4x - 2h\right) = -4x - 2(0)$$

$$= -4x$$

$f'(x) = -4x$, but $m = f'(0) = -4 \cdot 0 = 0$

$m = 0$

$(x_1, y_1) = (0, -8)$

$$y - (-8) = 0(x - 0)$$

$$y = -8$$