Lesson 6 : Basic Rules of differentiation

Recall : the derivative of f is

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

we denote the derivative with either

$f'(x)$ or $\frac{d}{dx}[f(x)]$

Constant rule:

let c be a number $\quad \frac{d}{dx}[c] = 0$

Power rule

let n be any nonzero number $\quad \frac{d}{dx}[x^n] = n x^{n-1}$

Constant multiple rule

let c be a number $\quad \frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}[f(x)]$

Sum rule

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$$

① $y = x^8 + 3x^4 + 2$, y'

$$y' = \frac{d}{dx}(y) = \frac{d}{dx}[x^8 + 3x^4 + 2]$$

$$= \frac{d}{dx}(x^8) + \frac{d}{dx}(3x^4) + \frac{d}{dx}(2)$$

$$= 8x^7 + 3\frac{d}{dx}(x^4) + 0$$

$= 8x^7 + 3 \cdot 4x^3$

$= 8x^7 + 12x^3$

(2) $y = 8\sqrt{x} - 2/x^2 + 3x^{\pi-1} + 100$, y'

$$y' = \frac{d}{dx}(y) = \frac{d}{dx}\left(8\sqrt{x} - \frac{2}{x^2} + 3x^{\pi-1} + 100\right)$$

$$= \frac{d}{dx}(8\sqrt{x}) + \frac{d}{dx}\left(\frac{-2}{x^2}\right) + \frac{d}{dx}\left(3x^{\pi-1}\right) + \frac{d}{dx}(100)$$

$$= 8\frac{d}{dx}(\underbrace{\sqrt{x}}_{= x^{1/2}}) - 2\frac{d}{dx}\left(\underbrace{\frac{1}{x^2}}_{= x^{-2}}\right) + 3\frac{d}{dx}\left(x^{\pi-1}\right) + 0$$

$\frac{d}{dx}(x^n) = n x^{n-1}$

$$= 8 \cdot \frac{1}{2}x^{1/2-1} - 2(-2)x^{-2-1} + 3(\pi-1)x^{\pi-1-1}$$

$$= 4x^{-1/2} + 4x^{-3} + 3(\pi-1)x^{\pi-2}$$

Derivative of sin / cos

$$\frac{d}{dx}(\sin x) = \cos x, \quad \frac{d}{dx}(\cos x) = -\sin x$$

(3) $f(x) = 5\cos x - 3\sin x$, $f'(x)$

$$f'(x) = \frac{d}{dx}\left(5\cos x - 3\sin x\right)$$

$$= \frac{d}{dx}(5\cos x) + \frac{d}{dx}(-3\sin x)$$

$$= 5\frac{d}{dx}(\cos x) - 3\frac{d}{dx}(\sin x)$$

$$= 5(-\sin x) - 3\cos x$$

$$= -5\sin x - 3\cos x$$

Derivative of (natural) exponentials

$$\frac{d}{dx}(e^x) = e^x$$

Very important!!

④ $f(x) = -5e^x + \sin x$, f'

$$f'(x) = \frac{d}{dx}(-5e^x + \sin x)$$

$$= \frac{d}{dx}(-5e^x) + \frac{d}{dx}(\sin x)$$

$$= -5\frac{d}{dx}(e^x) + \cos x$$

$$= -5e^x + \cos x$$

Lesson 6: Basic rules of differentiation

Derivative of $f(x)$ is

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

we denote this as $f'(x)$ or $\frac{d}{dx}\left(f(x)\right)$

Constant rule

let c be a number $\qquad \frac{d}{dx}(c) = 0$

Power rule

let n be a nonzero number, $\qquad \frac{d}{dx}\left(x^n\right) = nx^{n-1}$

Constant multiple rule

let c be a number $\qquad \frac{d}{dx}\left(c \cdot f(x)\right) = c \cdot \frac{d}{dx}\left(f(x)\right)$

Sum rule

$$\frac{d}{dx}\left(f(x) + g(x)\right) = \frac{d}{dx}\left(f(x)\right) + \frac{d}{dx}\left(g(x)\right)$$

1 $y = x^8 + 3x^4 + 2$, $\quad y'$

$$y' = \frac{d}{dx}(y) = \frac{d}{dx}\left(x^8 + 3x^4 + 2\right)$$

$$= \frac{d}{dx}\left(x^8\right) + \frac{d}{dx}\left(3x^4\right) + \frac{d}{dx}(2)$$

$$= 8x^{8-1} + 3\frac{d}{dx}(x^4) + 0$$

$$= 8x^7 + 3\cdot 4x^{4-1}$$

$$= 8x^7 + 12x^3$$

(2) $y = 8\sqrt{x} - 2/x^2 + 3x^{\pi-1} + 100$, y'

$$y' = \frac{d}{dx}(y) = \frac{d}{dx}\left(8\sqrt{x} - 2/x^2 + 3x^{\pi-1} + 100\right)$$

$$= \frac{d}{dx}(8\sqrt{x}) + \frac{d}{dx}\left(-2/x^2\right) + \frac{d}{dx}(3x^{\pi-1}) + \frac{d}{dx}(100)$$

$$= 8\frac{d}{dx}(\underbrace{\sqrt{x}}_{\text{"}x^{1/2}}) - 2\frac{d}{dx}\left(\underbrace{1/x^2}_{\text{"}x^{-2}}\right) + 3\frac{d}{dx}(x^{\pi-1}) + \frac{d}{dx}(100)$$

$$= 8\cdot\frac{1}{2}x^{1/2-1} - 2(-2)x^{-2-1} + 3(\pi-1)x^{\pi-1-1} + 0$$

$$= 4x^{-1/2} + 4x^{-3} + 3(\pi-1)x^{\pi-2}$$

Derivative of sin / cos

$$\frac{d}{dx}(\sin x) = \cos x, \quad \frac{d}{dx}(\cos x) = -\sin x$$

(3) $f(x) = 5\cos x - 3\sin x$, $f'(x)$

$$f'(x) = \frac{d}{dx}(5\cos x - 3\sin x)$$

$$= \frac{d}{dx}(5\cos x) + \frac{d}{dx}(-3\sin x)$$

$$= 5\frac{d}{dx}(\cos x) - 3\frac{d}{dx}(\sin x)$$

$$= 5(-\sin x) - 3(\cos x)$$

$$= -5\sin x - 3\cos x$$

Derivative of (natural) exponential

$$\frac{d}{dx}(e^x) = e^x$$

Very Important

④ $y = -5e^x + \sin x$, y'

$$y' = \frac{d}{dx}(-5e^x + \sin x)$$

$$= \frac{d}{dx}(-5e^x) + \frac{d}{dx}(\sin x)$$

$$= -5\frac{d}{dx}(e^x) + \cos x$$

$$= -5e^x + \cos x$$

⑤ $g(x) = \dfrac{10x + \sqrt{x}}{x^2} = \dfrac{10x}{x^2} + \dfrac{\sqrt{x}}{x^2}$

$$= \frac{10}{x} + x^{1/2 - 2}$$

$$= 10x^{-1} + x^{-3/2}$$