HW 6 # 9

$f(x) = \frac{x^4}{8} - \frac{16}{x^2}$ find tan line at $x = 2$

$$f'(x) = \frac{d}{dx}\left(\frac{x^4}{8} - \frac{16}{x^2}\right)$$

$$= \frac{d}{dx}\left(\frac{x^4}{8}\right) - \frac{d}{dx}\left(\frac{16}{x^2}\right)$$

$$\left\{ \frac{d}{dx}(x^n) = n x^{n-1} \right.$$

$$= \frac{1}{8} \cdot 4 \cdot x^{4-1} - 16 \frac{d}{dx}(x^{-2})$$

$$= \frac{1}{2} x^3 - 16(-2) x^{-2-1}$$

$$= \frac{1}{2} x^3 + 32 x^{-3}$$

Slope of tan line

$$f'(2) = \frac{1}{2}(2)^3 + 32(2^{-3}) = 4 + 4 = 8$$

to get y-value $f(2) = \frac{(2)^4}{8} - \frac{16}{(2)^2} = 2 - 4 = -2$

$$y - y_1 = m(x - x_1)$$

$$y + 2 = 8(x - 2)$$

$$y = 8(x-2) - 2$$
$$= 8x - 18$$

Lesson 7: Instantaneous rate of change

Recall: The derivative of $f(x)$, $\frac{d}{dx}(f)$ or $f'(x)$, measures the "slope" of $f(x)$

"rate of change"

$f'(c)$ = instantaneous rate of change at $x=c$

(1) Let $s(t) = 8t^2 + 3t - 1$ this represent the position of an object at time t.

Find the velocity.

Velocity = rate of change of position

= derivative of position

$$v(t) = \frac{d}{dt}\left(s(t)\right)$$

$$= \frac{d}{dt}\left(8t^2 + 3t - 1\right)$$

$$= \frac{d}{dt}(8t^2) + \frac{d}{dt}(3t^1) - \frac{d}{dt}(1)$$

$$= 8 \cdot 2t^{2-1} + 3 \cdot 1 \cdot t^{1-1} - 0$$

$$= 16t + 3$$

(2) Let $P(a) = \cos a + \frac{1}{3}a^3$ represent profits earned in thousands of \$ where a is money spent on ads. in thousands \$

What is R.o.C. of profit.

find derivative $P(a)$.

$$P'(a) = \frac{d}{da}(\cos a) + \frac{d}{da}\left(\frac{1}{3}a^3\right)$$

$$= -\sin a + \frac{1}{3} \cdot 3a^{3-1}$$

$$= -\sin a + a^2$$

(3) Suppose that G = ammount of grit

C = love for calculus

$$C = 3G + 10$$

a) What is R.o.C. of C with respect to G

derivative of C G is the variable

$$\frac{d}{dG}(C) = \frac{d}{dG}(3G+10) = \frac{d}{dG}(3G) + \frac{d}{dG}(10)$$

$$= 3 + 0 = 3$$

B) What is the R.o.C of G w.r.t. C

derivative of G   C is the variable

$C = 3G + 10$

Solve for G:

$C - 10 = 3G$

$\frac{C-10}{3} = G$

$G = \frac{1}{3}C - \frac{10}{3}$

$$\frac{d}{dC}(G) = \frac{d}{dC}\left(\frac{1}{3}C - \frac{10}{3}\right)$$

$$= \frac{d}{dC}\left(\frac{1}{3}C^{1}\right) - \frac{d}{dC}\left(\frac{10}{3}\right)$$

$$= \frac{1}{3} + \bigcirc$$

$$= 1/3$$

(4) The pop. of a city since the year 2000 is

$P(t) = 800t^2 - t + 100$

where t is number of years since 2000.

In what year is R.o.C of pop. equal to 3199.

derivative of P

$$\frac{d}{dt}(800t^2 - t + 100) = \frac{d}{dt}(800t^2) - \frac{d}{dt}(t^1) + \frac{d}{dt}(100)$$

$$= 800 \cdot 2t^{2-1} - 1 \cdot t^{1-1} + 0$$

$$P'(t) = 1600t - 1$$

$$3199 = 1600t - 1$$

$$3200 = 1600t \qquad \frac{3200}{1600} = 2$$

$$t = 2$$

Pop. w/ be increasing w/ R.o.C. of 3199 in the year 2002

What is R.o.C. of area of circle w.r.t. it radius r ?

$$\begin{cases} A = \pi r^2 \\ A = \text{area} \\ r = \text{radius} \end{cases}$$

HW 6 # 9

$f(x) = \frac{x^4}{8} - \frac{16}{x^2}$

find tan. line to f at $x = 2$

$$f'(x) = \frac{d}{dx}\left(\frac{x^4}{8} - \frac{16}{x^2} \right)$$

$\parallel$ $16x^{-2}$

$$= \frac{d}{dx}\left(\frac{x^4}{8}\right) - \frac{d}{dx}\left(\frac{16}{x^2}\right)$$

$$= \frac{1}{8}\cdot 4x^{4-1} - 16(-2)x^{-2-1}$$

$$= \frac{1}{2}x^3 + 32x^{-3}$$

$$\underline{y - y_1 = m(x - x_1)}$$

$$m = f'(2)$$

$$= \frac{1}{2}(2)^3 + 32(2)^{-3}$$

$$= 4 + 4$$

$$= \underline{8}$$

$$y_1 = f(2) = \frac{(2)^4}{8} - \frac{16}{(2)^2} = 2 - 4 = -2$$

$$y - (-2) = 8(x - 2)$$

$$y = 8(x-2) - 2 \checkmark$$

Lesson 7: Instantaneous rate of change

Recall: the derivative of f measure

the "slope" of f

another "rate of change" of f

= derivative of f

$f'(c)$ = instantaneous rate of change of f at c

(1) Let $s(t) = 8t^2 + 3t - 1$ represent the position of an object at time t.

Find velocity function.

velocity = R. o. C. of position

= derivative of position

Velocity function be $v(t)$

$$v(t) = \frac{d}{dt}\left(s(t)\right) = \frac{d}{dt}\left(8t^2 + 3t - 1\right)$$

$$= \frac{d}{dt}\left(8t^2\right) + \frac{d}{dt}\left(3t^1\right) - \frac{d}{dt}(-1)$$

$$= 8 \cdot 2 \cdot t^{2-1} + 3 \cdot 1 \cdot t^{1-1} +$$

$= 16t + 3$

What is velocity at $t = 10$

$\underline{v(10) = 16(10) + 3 = 163 \text{ m/s}}$

(2) $P(a) = \cos a + \frac{1}{3}a^3$

$P(a)$: represents profit thousands of \$

a : represent money spent on ads thousands of \$

What is $\underline{\text{R.o.C. of profit.}}$

derivative of P

$$P'(a) = \frac{d}{da}\left(\cos a + \frac{1}{3}a^3\right)$$

$$= \frac{d}{da}(\cos a) + \frac{d}{da}\left(\frac{1}{3}a^3\right)$$

$$= -\sin a + \frac{1}{3} \cdot 3a^{3-1}$$

$$= -\sin a + a^2$$

(3) The pop. of a city is

$$P(t) = 800t^2 - t + 100$$

where t is number of years since year 2000

In which year is the pop. increasing at a R.o.C. of 3199?

$$P'(t) = \frac{d}{dt}\left(800t^2 - t + 100\right)$$

$$= \frac{d}{dt}\left(800t^2\right) + \frac{d}{dt}\left(-t^1\right) + \frac{d}{dt}\left(100\right)$$

$$= 800 \cdot 2 \cdot t^{2-1} - 1 \cdot t^{1-1} +$$

$$= 1600t - 1$$

What value of t does $1600t - 1 = 3199$?

$$1600t = 3200$$

$$t = 2$$

In the year 2002 the pop. has R.o.C of 3199.

(4) Suppose G = ammount of grit
C = love for calculus

$$C = 3G + 10$$

a) What is R.o.C. of C w.r.t. G

derivative of C   G is the variable

$$\frac{d}{dG}(C) = \frac{d}{dG}(3G+10) = \frac{d}{dG}(3G) + \frac{d}{dG}(10)$$

$$= 3 + 0 = 3$$

b) What is the R.o.C. of G w.r.t. C?

derivative of G   C is variable

$$C = 3G + 10$$
$$3G = C - 10$$
$$G = \frac{C-10}{3}$$
$$G = \frac{1}{3}C - \frac{10}{3}$$

$$\frac{d}{dC}(G) = \frac{d}{dC}\left(\frac{1}{3}C - \frac{10}{3}\right)$$

$$= \frac{d}{dC}\left(\frac{1}{3}C^{1}\right) + \frac{d}{dC}\left(\frac{10}{3}\right)$$

$$= \frac{1}{3}\cdot 1 \cdot C^{1-1} + 0$$

$$= \frac{1}{3}$$

5 What is R.o.C. of the area of a square with side length s?

$A = s^2$ area of square

$A' = 2s^{2-1} = 2s$

Area of circle

$A = \pi r^2$

r = radius