HW 7 #8 | Weight of a girl

$w(t) = 0.00001t^3 - 0.006t^2 + 1.004t + 6.5$

t : months

R.o.C. of weight when she is 3.5 years old?

derivative of w — variable t

$w'(t) = 0.000003t^2 - 0.012t + 1.004$

3.5 years $= 3 \times 12 + \frac{1}{2} \cdot 12 = 42$ months

$w'(42) = ?$

Lesson 8 : Product rule

$$\frac{d}{dx}\left[f(x)\, g(x) \right] = f'(x)\, g(x) + f(x)\, g'(x)$$

$$\frac{d}{dx}\left(f(x) \cdot g(x) \right) \neq f'(x)\, g'(x)$$

Very bad!!

① $f(x) = \underbrace{(x^{16} + 3x + 5)}_{h(x)}\underbrace{(2x^{-5} + x + 3)}_{g(x)}$

$$f'(x) = (16x^{15} + 3)(2x^{-5} + x + 3) + (x^{16} + 3x + 5)(-10x^{-6} + 1)$$

✓

$$\frac{d}{dx}(f(x)) = h'(x)g(x) + h(x)g'(x)$$

② $y = \underbrace{6\sqrt[4]{x^3}}_{h(x)}\underbrace{(1 + 3x)}_{g(x)}$ $\qquad y'(4) = ?$

$$\sqrt[4]{x^3} = x^{3/4}$$

$$y' = h'(x)g(x) + h(x)g'(x)$$

$$= \left(6 \cdot \frac{3}{4} x^{3/4 - 1}\right)(1 + 3x) + \left(6x^{3/4}\right)(0 + 3)$$

$$= \frac{9}{2} x^{-1/4} * (1 + 3x) + 18x^{3/4}$$ ✓

$$y'(4) = \frac{9}{2}(4)^{-1/4}(1 + 3 \cdot 4) + 18(4)^{3/4}$$

$$= \frac{9}{2}\left(\frac{1}{\sqrt{2}}\right)(13) + 18(2\sqrt{2})$$

$$= \frac{117}{2\sqrt{2}} + 36\sqrt{2}$$ ✓

(3) $y = 3e^x \sin x + x\cos x$

$$y' = (3e^x)(\sin x) + (3e^x)(\cos x)$$
$$(1)(\cos x) + (x)(-\sin x)$$

$$y' = 3e^x \sin x + 3e^x \cos x + \cos x - x\sin x \quad ✓$$

(4) find all x such that $y = 8x^6 e^x$ has a horizontal tangent line.

derivative

derivative $= 0$. 0 slope we are flat!

$$y' = (8 \cdot 6x^5)(e^x) + (8x^6)e^x$$
$$= e^x(48x^5 + 8x^6)$$
$$= 8x^5 e^x(6 + x)$$

$$0 = 8x^5 \cancel{e^x}(6 + x)$$

$x = 0$ $x = -6$ ✓

HW 7 #7

$$C = \frac{5}{9}(F - 32)$$

a) R.o.C. of C w.r.t F

derivative of C   F is variable

$$\frac{d}{dF}(C) = \frac{d}{dF}\left(\frac{5}{9}(F-32)\right)$$

$$= \frac{5}{9}\frac{d}{dF}(F-32) = \frac{5}{9}(1-0)$$

$$= \frac{5}{9}$$

b) R.o.C. of F w.r.t C

derivative of F   C is variable

$$C = \frac{5}{9}(F-32)$$

$$\frac{9}{5}C = F - 32$$

$$F = \frac{9}{5}C + 32$$

$$\frac{d}{dC}(F) = \frac{d}{dC}\left(\frac{9}{5}C + 32\right)$$

$$= \frac{9}{5} + 0$$

$$= 9/5 \quad \checkmark$$

Lesson 8: Product rule

$$\frac{d}{dx}\left(h(x)\cdot g(x)\right) = h'(x)\,g(x) + h(x)\cdot g'(x)$$

$$\frac{d}{dx}\left(h(x)\,g(x)\right) \neq h'(x)\,g'(x)$$

Very bad!!

① $f(x) = \underbrace{(x^{16}+3x+5)}_{h(x)}\underbrace{(2x^{-5}+x+3)}_{g(x)}$

$f'(x) = h'(x)\,g(x) + h(x)\,g'(x)$ ← product rule

$$= (16x^{15}+3+0)(2x^{-5}+x+3)$$

$$+ (x^{16}+3x+5)(-10x^{-6}+1+0)$$

$$= (16x^{15}+3)(2x^{-5}+x+3) + (x^{16}+3x+5)(-10x^{-6}+1)$$

② $y = \underbrace{6\sqrt[4]{x^3}}_{h(x)} \underbrace{(1+3x)}_{g(x)}$ $\quad y'(4) = ?$

$6\sqrt[4]{x^3} = 6x^{3/4}$

$y' = h'(x)g(x) + h(x)g'(x)$ ← product rule

$= \left(6 \cdot \frac{3}{4} x^{3/4 - 1}\right)(1+3x) + 6x^{3/4}(0+3)$

$= \frac{9}{2} x^{-1/4}(1+3x) + 18x^{3/4}$

$y'(4) = \frac{9}{2}(4)^{-1/4}(1+3\cdot 4) + 18(4)^{3/4}$

$= \frac{9}{2}\left(\frac{1}{\sqrt{2}}\right) \cdot 13 + 18(2\sqrt{2})$

$= \frac{117}{2\sqrt{2}} + 36\sqrt{2}$ ✓

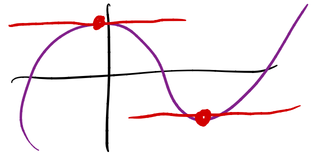
③ $y = 3e^x \sin x + x\cos x$, $\quad y' = ?$

$y' = (3e^x)(\sin x) + (3e^x)(\cos x)$

$+ (1)(\cos x) + x(-\sin x)$

$= 3e^x \sin x + 3e^x \cos x + \cos x - x\sin x$ ✓

(4) find all x such that $y = 8x^6 e^x$ has horizontal tangent line.

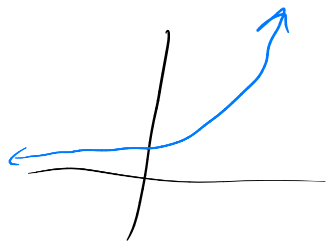


$f'(c)$ = slope of tangent line $= 0$

$$y = 8x^6 e^x$$

$$y' = (8 \cdot 6x^5)(e^x) + 8x^6 e^x$$

$$= 8x^5 e^x (6 + x)$$



$$0 = 8x^5 \cancel{e^x} (6 + x)$$

$$\downarrow \qquad\qquad \downarrow$$

$$x = 0 \qquad\qquad x = -6$$

$x = 0, -6$ have horizontal tangent lines.