$$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}\left(x^{-1}\right) = -1 \cdot x^{-2} = -x^{-2} = \frac{-1}{x^2}$$

HW 8 #10 $y = 6x \sin x$ find tangent line at $x = \pi$

$g(x) = 6x$, $h(x) = \sin x$

point-slope form of a line

$$y - y_1 = m(x - x_1)$$

m: slope ✓

(x_1, y_1): is a point on the line ✓

$$y' = g'(x)\,h(x) + g(x)\,h'(x)$$

$$= 6(\sin x) + 6x(\cos x)$$

$$= 6\sin x + 6x\cos x$$

$$m = y'(\pi) = 6\sin(\pi) + 6\pi\cos(\pi) = -6\pi$$

$$y_1 = 6(\pi)\sin(\pi) = 0$$

$$y - 0 = -6\pi(x - \pi)$$

$$y = -6\pi(x - \pi)$$ ✓

Lesson 9: The quotient rule; Derivatives of other trig functions

The quotient rule

$$\frac{d}{dx}\left(\frac{g(x)}{h(x)}\right) = \frac{g'(x)h(x) - g(x)h'(x)}{(h(x))^2}$$

① $$f(x) = \frac{x^3}{x^2+3x+1} \quad \begin{matrix} g(x) \\ h(x) \end{matrix}$$

$$f'(x) = \frac{3x^2(x^2+3x+1) - x^3(2x+3)}{(x^2+3x+1)^2}$$

$$= \frac{x^2\left(3(x^2+3x+1) - x(2x+3)\right)}{(x^2+3x+1)^2}$$

$$= \frac{x^2(3x^2+9x+3-2x^2-3x)}{(x^2+3x+1)^2}$$

$$= \frac{x^2(x^2+6x+3)}{(x^2+3x+1)^2}$$ ✓

(2) $f(x) = \tan x = \dfrac{\sin x}{\cos x}$ $\quad \sin x = g(x)$, $\cos x = h(x)$

$g'(x) = \cos x$

$h'(x) = -\sin x$

$$f'(x) = \frac{g'(x)\,h(x) - g(x)\,h'(x)}{(h(x))^2}$$

$$= \frac{\cos x \cos x - \sin x(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

Derivatives of trig funtions

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

All of these are applications of quotient rule

(3) $y = \underbrace{-3e^x}_{g(x)}\ \underbrace{\sec x}_{h(x)}$

$$y' = g'(x)\,h(x) + g(x)\,h'(x)$$

$$= -3e^x \sec x + (-3e^x)(\sec x \tan x)$$

$$= -3e^x \sec x\,(1 + \tan x)$$

(4) Let a be a constant,

$$f(x) = \frac{(a+x^2)}{a \sin x} \quad \begin{matrix} = g(x) \\ = h(x) \end{matrix}, \qquad f'(x) = ?$$

$$f'(x) = \frac{g'(x)\,h(x) - g(x)\,h'(x)}{(h(x))^2}$$

$g'(x) = 2x$

$h'(x) = a\cos x$

$$= \frac{(2x)(a\sin x) - (a+x^2)(a\cos x)}{a^2 \sin^2 x}$$

(5) $$f(x) = \frac{x^2 + \sqrt[3]{x}}{1-x} \quad \begin{matrix} = g(x) \\ = h(x) \end{matrix}$$

$$f'(x) = \frac{g'(x)\,h(x) - g(x)\,h'(x)}{(h(x))^2}$$

$g(x) = x^2 + \sqrt[3]{x}$

$\quad = x^2 + x^{1/3}$

$h(x) = 1 - x$

$h'(x) = -1$

$g'(x) = 2x + \frac{1}{3}x^{-2/3}$

$$f'(x) = \frac{(2x + \frac{1}{3}x^{-2/3})(1-x) - (x^2 + \sqrt[3]{x})(-1)}{(1-x)^2}$$

$$= \frac{(2x + \frac{1}{3}x^{-2/3})(1-x) + (x^2 + \sqrt[3]{x})}{(1-x)^2}$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}\left(x^{-1}\right) = -1 \cdot x^{-1-1} = -x^{-2} = \frac{-1}{x^2}$$

HW 8 # 5

$$y = \underbrace{6\sin x}_{g(x)} \underbrace{(9\cos x + 3\sin x)}_{h(x)}$$

$$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6}} = ??$$

$$\frac{dy}{dx} = y' = g'(x)\,h(x) + g(x)\,h'(x)$$

$$= 6\cos x\,(9\cos x + 3\sin x) + 6\sin x\,(-9\sin x + 3\cos x)$$

$$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6}} = y'\left(\frac{\pi}{6}\right) = 6\cos\left(\frac{\pi}{6}\right)\left(9\cos\frac{\pi}{6} + 3\sin\frac{\pi}{6}\right)$$

$$+ 6\sin\left(\frac{\pi}{6}\right)\left(-9\sin\frac{\pi}{6} + 3\cos\frac{\pi}{6}\right)$$

Lesson 9: The quotient rule; Derivatives of other trig functions,

The quotient rule

$$\frac{d}{dx}\left(\frac{g(x)}{h(x)}\right) = \frac{g'(x)h(x) - g(x)h'(x)}{(h(x))^2}$$

① $f(x) = \dfrac{x^3}{x^2+3x+1}$ $\quad x^3 = g(x)$, $\quad x^2+3x+1 = h(x)$

$$f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{(h(x))^2}$$

$g(x) = x^3$

$g'(x) = 3x^2$

$h(x) = x^2+3x+1$

$h'(x) = 2x+3$

$$= \frac{(3x^2)(x^2+3x+1) - x^3(2x+3)}{(x^2+3x+1)^2}$$

$$= \frac{x^2\left(3(x^2+3x+1) - x(2x+3)\right)}{(x^2+3x+1)^2}$$

$$= \frac{x^2(3x^2+9x+3-2x^2-3x)}{(x^2+3x+1)^2}$$

$$= \frac{x^2(x^2+6x+3)}{(x^2+3x+1)^2}$$

(2) $f(x) = \tan x = \frac{\sin x}{\cos x}$ $\sin x = g(x)$, $\cos x = h(x)$

$$\frac{df}{dx} = \frac{g'(x)h(x) - g(x)h'(x)}{(h(x))^2}$$

$g(x) = \sin x$

$g'(x) = \cos x$

$h(x) = \cos x$

$h'(x) = -\sin x$

$$= \frac{\cos x \cos x - \sin x(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \qquad (\cos^2 x + \sin^2 x = 1)$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

Derivatives of Trig functions

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

Applications of quotient rule.

(3) $y = \underbrace{-3e^x}_{g(x)} \underbrace{\sec x}_{h(x)}$, $dy/dx = ?$

$$dy/dx = g'(x)h(x) + g(x)h'(x)$$

$$= (-3e^x)\sec x + (-3e^x)(\sec x \tan x)$$

$= -3e^x \sec x (1 + \tan x)$

(4) $f(x) = \dfrac{x^2 + \sqrt[3]{x}}{1 - x}$ $\quad x^2 + \sqrt[3]{x} = g(x)$, $\quad 1 - x = h(x)$

$$\frac{df}{dx} = \frac{g'(x)\,h(x) - g(x)\,h'(x)}{(h(x))^2}$$

$g(x) = x^2 + \sqrt[3]{x}$

$= x^2 + x^{1/3}$

$g'(x) = 2x + \frac{1}{3}x^{-2/3}$

$h(x) = 1 - x$

$h'(x) = -1$

$$\frac{df}{dx} = \frac{(2x + \frac{1}{3}x^{-2/3})(1-x) - (x^2 + x^{1/3})(-1)}{(1-x)^2}$$

$$= \frac{(2x + \frac{1}{3}x^{-2/3})(1-x) + (x^2 + x^{1/3})}{(1-x)^2}$$

yay! done!

(5) Let a be a constant

Let $f(x) = \dfrac{a + x^2}{a \sin x}$ $\quad a + x^2 = g(x)$, $\quad a\sin x = h(x)$, $\quad df/dx = ?$

$$\frac{df}{dx} = \frac{g'(x)\,h(x) - g(x)\,h'(x)}{(h(x))^2}$$

$g(x) = a + x^2$

$g'(x) = 0 + 2x = 2x$

$h(x) = a \sin x$

$h'(x) = a \cos x$

$$\frac{df}{dx} = \frac{(2x)(a \sin x) - (a + x^2)(a \cos x)}{(a \sin x)^2}$$