Quiz 4

June Weiland

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Problem 1. Differentiate $y = 3\sec(10x^3 - 2x^2 + 1)$.

Solution: Let $g(x) = 3\sec x$ and $h(x) = 10x^3 - 2x^2 + 1$, then $y = g(h(x))$. By the chain rule $y' = g'(h(x)) \cdot h'(x)$. Since $g'(x) = 3\sec x \tan x$ and $h'(x) = 30x^2 - 4x$, then

$$y' = 3\sec(h(x))\tan(h(x)) \cdot h'(x) = 3\sec(10x^3 - 2x^2 + 1)\tan(10x^3 - 2x^2 + 1)(30x^2 - 4x).$$

Problem 2. Find the second derivative of $g(x) = x^5 \ln(2x)$.

Solution: By the product rule,

$$\begin{aligned}
g'(x) &= \frac{d}{dx}(x^5) \cdot \ln(2x) + x^5 \cdot \frac{d}{dx}(\ln(2x)) \\
&= 5x^4 \ln(2x) + x^5 \cdot \frac{2}{2x} \\
&= 5x^4 \ln(2x) + x^4.
\end{aligned}$$

Then

$$\begin{aligned}
g''(x) &= \frac{d}{dx}(5x^4) \cdot \ln(2x) + 5x^4 \cdot \frac{d}{dx}(\ln(2x)) + 4x^3 \\
&= 20x^3 \ln(2x) + 5x^3 + 4x^3 \\
&= 20x^3 \ln(2x) + 9x^3.
\end{aligned}$$