Quiz 6

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Problem 1. Consider the function $y = x^4 + 2x^3 - 12x^2 + 12x - 36$.

(a) Find the largest open intervals on which the function is concave up.

(b) Find the largest open intervals on which the function is concave down.

(c) Find all inflection points.

Solution: In order to determine the concavity of y, we will need to know when $y'' = 0$ or when y'' does not exist. In either case we will need the second derivative of y,

$$y' = 4x^3 + 6x^2 - 24x + 12$$

and

$$y'' = 12x^2 + 12x - 24 = 12(x+2)(x-1).$$

Since y'' always exists, then we are only concerned about the case where $y'' = 0$, i.e., when

$$0 = 12(x+2)(x-1).$$

Hence we see that at $x = -2$ and $x = 1$ we have that $y'' = 0$. This leaves us with three intervals to check the concavity of y; namely; $(-\infty, -2)$, $(-2, 1)$ and $(1, \infty)$. Note that it suffices to check the concavity for a single x value in each of the intervals. Since

$$y''(-3) = 12(-3+2)(-3-1) > 0,$$

$$y''(0) = 12(0+2)(0-1) < 0$$

and

$$y''(2) = 12(2+2)(2-1) > 0,$$

then we are concave up on the interval $(-\infty, -2) \cup (1, \infty)$ and concave down on the interval $(-2, 1)$. Finally near $x = -2$, y'' switches from positive to negative so this is the x-value of an inflection point. Similarly, near $x = 1$, y'' switches from negative to positive so this is the x-value of an inflection point. To obtain the corresponding y-values for the inflection points we evaluate:

$$y(-2) = (-2)^4 + 2(-2)^3 - 12(-2)^2 + 12(-2) - 36 = -108$$

and

$$y(1) = (1)^4 + 2(1)^3 - 12(1)^2 + 12(1) - 36 = -33.$$

Therefore $(-2, -108)$ and $(1, -33)$ and the inflection points for y