Quiz 7

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Problem 1. Consider the function
$$f(x) = \frac{3x^2 + 2x + 1}{x^2 - 1}.$$

(a) Find the vertical asymptote(s).

(b) Compute
$$\lim_{x \to \infty} f(x).$$

(c) Compute
$$\lim_{x \to -\infty} f(x).$$

(d) Find the horizontal aysmptote, if it exists.

(e) Does $f(x)$ have a slant asymptote? *Hint: look at the degree.*

Solution:

(a) The vertical asymptotes will occur when the denominator of f is equal to 0, i.e., when $x^2 - 1 = 0$. Hence the vertical asymptotes are at $x = \pm 1$.

(b) Since
$$f(x) = \frac{3x^2 + 2x + 1}{x^2 - 1} \sim \frac{3x^2}{x^2} = 3,$$
then $\lim_{x \to \infty} f(x) = 3$.

(c) By (b) we also have $\lim_{x \to -\infty} f(x) = 3$.

(d) Since (b) or (c) is finite and both are equal to 3, then we have a horizontal asymptote at $y = 3$.

(e) Since the degree of the numerator is the same as the degree of the denominator we will not have a slant asymptote.