Quiz 8

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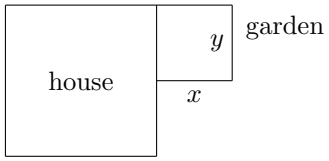
Problem 1. You have 100 feet of fence to make a rectangular vegetable garden alongside the wall of your house. The wall of the house bounds one side of the vegetable garden. What is the largest possible area of the vegetable garden?

Solution: Consider the following picture of the garden



Since we want to maximize the area of the garden our objective function will be

$$A = xy.$$

Since we only have 100 feet of fence the constraint will be

$$100 = 2x + y$$

as well as $x, y > 0$. If we solve the constraint equation for y, then we see that $y = 100 - 2x$. Hence

$$A = xy = x(100 - 2x) = 100x - 2x^2$$

is our new objective function. Since $y > 0$, then $100 - 2x > 0$ and whence $x < 50$. So our new constraint is $0 < x < 50$. Our task is now to find the absolute maximum of the function $A = 100x - 2x^2$ on the interval $(0, 50)$. The derivative is

$$A' = 100 - 4x$$

so the critical numbers occur when $0 = 100 - 4x$, i.e., when $x = 25$. This is the only critical number in our interval $(0, 50)$. In order to determine if it is an absolute maximum we need to know if it is a relative maximum. We will do this by using the second derivative test. Since

$$A'' = -4$$

is always negative, then $A''(25) < 0$ and hence $x = 25$ is the x value of our absolute maximum. Finally, the largest possible area of the vegetable garden will be

$$A(25) = (25)(100 - 2(25)) = 1250\text{ft}^2.$$