$\boxed{\text{HW 10 \# 6}}$ $y = \sqrt{7x^3 - 3x^2 - 7/x}$ $y' = ?$

$y = g(h(x))$ $y' = g'(h(x)) \cdot h'(x)$

$g(x) = \sqrt{x}$ " $x^{1/2}$ $h(x) = 7x^3 - 3x^2 - 7/x$ " $-7x^{-1}$

$g'(x) = \frac{1}{2} x^{-1/2}$ $h'(x) = 21x^2 - 6x + 7x^{-2}$

$$y' = \frac{1}{2}\left(7x^3 - 3x^2 - \frac{7}{x}\right)^{-1/2} \cdot \left(21x^2 - 6x + 7x^{-2}\right)$$

$$= \frac{1}{2} \cdot \frac{21x^2 - 6x + 7x^{-2}}{(7x^3 - 3x^2 - 7/x)^{1/2}}$$

$$= \frac{1}{2} \cdot \frac{21x^2 - 6x + 7x^{-2}}{\sqrt{7x^3 - 3x^2 - 7/x}}$$ ✓

Review for Exam # 1

① $f(x) = \begin{cases} \cos x & x \leq 0 \\ x+1 & 0 < x \leq 1 \\ -x & x > 1 \end{cases}$

f is cont at $x=c$ if
1) $f(c)$ is defined
2) $\lim_{x\to c} f(x)$ exists
3) $f(c) = \lim_{x\to c} f(x)$

find and classify all discont.

discont occur when we divide by 0 or bc. of piecewise.

possible discont. are $x=0$ and $x=1$

$\underline{x=0}$ 1) $f(0) = \cos(0) = 1$ ✓

2) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \cos x = \cos(0) = 1$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x+1) = 0+1 = 1$

$\lim_{x\to 0} f(x) = 1$ ✓

3) $f(0) = 1$ $\lim_{x\to 0} f(x) = 1$ ✓

f is cont. at $x=0$.

$\underline{x=1}$ 1) $f(1) = 1+1 = 2$ ✓

2) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x+1) = 1+1 = 2$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (-x) = -1$

$\lim_{x \to 1} f(x)$ DNE $\times$

So $x=1$ is a discontinuity

Since $\lim_{x \to 1^-} f(x) = 2$ and $\lim_{x \to 1^+} f(x) = -1$

then $x=1$ is a jump discont.

(2) $y = \underbrace{3x}_{h(x)} \underbrace{\sin x}_{g(x)}$ find the $\underbrace{\text{tangent line}}_{\text{derivative}}$ $\underbrace{\text{at } x=\pi}_{\text{at } \pi}$

$$y - y_1 = m(x - x_1)$$

m : slope = derivative at $x = \pi$

(x_1, y_1) : point on the line $(\pi, 3\pi\sin(\pi)) = (\pi, 0)$

$$y' = h'(x)g(x) + h(x)g'(x)$$

$$= 3\sin x + 3x\cos x$$

$$y'(\pi) = 3\sin(\pi) + 3\pi\cos(\pi)$$

$$= 0 - 3\pi$$

$$= -3\pi$$

$$y - 0 = -3\pi(x - \pi)$$

$$y = -3\pi(x - \pi)$$

③ Let $s(t) = \left(\frac{t^3+1}{\cos t}\right)^5$ be position of an object in meters and t seconds.

find the velocity function for this obj.

$v(t) =$ Velocity $=$ R.o.C. of position

$=$ derivative of position

$v(t) = s'(t) \qquad s(t) = \left(\frac{t^3+1}{\cos t}\right)^5$

$g(t) = t^5 \qquad h(t) = \frac{t^3+1}{\cos t}$

$g'(t) = 5t^4 \qquad h'(t) = \frac{3t^2(\cos t) - (t^3+1)(-\sin t)}{\cos^2 t}$

$= \frac{3t^2\cos t + (t^3+1)\sin t}{\cos^2 t}$

$s'(t) = g'(h(t)) \cdot h'(t)$

$v(t) = 5\left(\frac{t^3+1}{\cos t}\right)^4 \cdot \frac{3t^2\cos t + (t^3+1)\sin t}{\cos^2 t}$

④ $f(x) = \underbrace{3e^x}_{g(x)} \underbrace{\sec x}_{h(x)}$ find tangent line at $x=0$

slope tan line = derivative evaluated at 0

$y - y_1 = m(x - x_1)$ $\quad y - 3 = 3(x - 0)$ ✓

m: slope = derivative of f at 0

$(x_1, y_1) = (0, f(0)) = (0, 3e^0 \sec(0))$

$= (0, 3)$

$f'(x) = g'(x)h(x) + g(x)h'(x)$

$= 3e^x \sec x + 3e^x \sec x \tan x$

$f'(0) = 3e^0 \sec(0) + 3e^0 \sec(0) \tan(0)$

$= 3 + 3 \cdot 1 \cdot 1 \cdot 0$

$= 3$

HW 10 # 7

$$y = \frac{3}{(3 - x^6)^{3/2}} \qquad y' = ?$$

$$y = 3(3 - x^6)^{-3/2}$$

$$g(x) = 3x^{-3/2} \qquad h(x) = 3 - x^6$$

$$g'(x) = 3 \cdot \frac{-3}{2} \cdot x^{-3/2 - 1} \qquad h'(x) = -6x^5$$

$$= \frac{-9}{2} x^{-5/2}$$

$$y' = g'(h(x)) \cdot h'(x)$$

$$= \frac{-9}{2}(3 - x^6)^{-5/2} \cdot (-6x^5)$$

$$= 27x^5(3 - x^6)^{-5/2}$$

Review for Exam #2

① $f(x) = \begin{cases} \cos x & x \le 0 \\ x+1 & 0 < x \le 1 \\ -x & x > 1 \end{cases}$ find / classify all discont.

discont. occur when we divide by zero or when we have a piecewise function.

f is cont. at c if

1) $f(c)$ is defined

2) $\lim_{x\to c} f(x)$ exists

3) $f(c) = \lim_{x\to c} f(x)$

So only possible discont. are at $x=0$ and $x=1$.

$\underline{x=0}$ 1) $f(0) = \cos(0) = 1$ ✓

2) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \cos x = \cos(0) = 1$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x+1) = 0+1 = 1$

$\lim_{x\to 0} f(x) = 1$ ✓

3) $f(0) = 1 = \lim_{x\to 0} f(x)$ ✓

So f is cont. at $x=0$

$\underline{x=1}$ 1) $f(1) = 1+1 = 2$ ✓

2) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x+1) = 1+1 = 2$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (-x) = -1$

So $\lim_{x\to 1} f(x)$ DNE. ✗

So $x = 1$ is a discont. of f.

Since $\lim_{x\to 1^-} f(x) = 2$ and $\lim_{x\to 1^+} f(x) = -1$,

then $x = 1$ is a jump discont.

(2) $y = \underbrace{3x}_{h(x)}\, \underbrace{\sin x}_{g(x)}$ find the tangent line at $x = \pi$

$y - y_1 = m(x - x_1)$ slope of tan line = derivative at $x = \pi$

m: slope of tan line $= y'(\pi)$

(x_1, y_1): point on tan line $(\pi, y(\pi))$

$y(\pi) = 3\pi \sin\pi = 0$

$y' = h'(x)\, g(x) + h(x)\, g'(x)$

$= 3 \sin x + 3x \cos x$

$y'(\pi) = 3\sin(\pi) + 3\pi \cos\pi = 3\cdot 0 + 3\pi(-1) = -3\pi$

$y - 0 = -3\pi(x - \pi)$

$y = -3\pi(x - \pi)$ ✓

(3) Let $s(t) = \left(\frac{t^3+1}{\cos t} \right)^5$ represent the position of an obj in meters. Let t represent time in seconds. Find the velocity function of the obj.

$v(t)$ = velocity function = R.o.C. of position

= derivative of $s(t)$

$= s'(t)$

$$s(t) = \left(\frac{t^3+1}{\cos t} \right)^5$$

$$g(t) = t^5 \qquad h(t) = \frac{t^3+1}{\cos t}$$

$$g'(t) = 5t^4 \qquad h'(t) = \frac{3t^2\cos t - (t^3+1)(-\sin t)}{\cos^2 t}$$

$$= \frac{3t^2\cos t + (t^3+1)\sin t}{\cos^2 t}$$

$$s'(t) = g'(h(t)) \cdot h'(t)$$

$$v(t) = 5\left(\frac{t^3+1}{\cos t} \right)^4 \cdot \frac{3t^2\cos t + (t^3+1)\sin t}{\cos^2 t}$$