# Review for Exam # 2 : 11-19

**Chain rule** $y = f(g(x))$

$$\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$$

**Implicit differentiation**

$$y' = \frac{dy}{dx} = \frac{d}{dx}[y]$$

① $6\tan(x/y) = 13x$ find $dy/dx$

$$\frac{d}{dx}\left[6\tan\left(\frac{x}{y}\right)\right] = \frac{d}{dx}[13x]$$

$$6\sec^2\left(\frac{x}{y}\right) \cdot \frac{d}{dx}\left[\frac{x}{y}\right] = 13$$

$$6\sec^2\left(\frac{x}{y}\right) \cdot \frac{1 \cdot y - x \cdot y'}{y^2} = 13$$

$$\left\{ \frac{1}{\sec x} = \cos x \right.$$

$$\frac{y - xy'}{y^2} = \frac{13}{6}\cos^2\left(\frac{x}{y}\right)$$

$$y - xy' = \frac{13}{6} y^2 \cos^2(x/y)$$

$$xy' = y - \frac{13}{6} y^2 \cos^2(x/y)$$

$$\frac{dy}{dx} = y' = y/x - \frac{13}{6} y^2/x \cos^2(x/y). \quad \checkmark$$

(2) $2\sin(6x+7y) = 5xy$ find $dy/dx$

$$\frac{d}{dx}\left[2\sin(6x+7y)\right] = \frac{d}{dx}\left[5xy\right]$$

$$2\cos(6x+7y)\cdot(6+7y') = 5\cdot y + 5x\cdot y'$$

$$12\cos(6x+7y) + 14\cos(6x+7y)\cdot y' = 5y + 5xy'$$

$$14\cos(6x+7y)\,y' - 5xy' = 5y - 12\cos(6x+7y)$$

$$(14\cos(6x+7y) - 5x)\,y' = 5y - 12\cos(6x+7y)$$

$$y' = \frac{5y - 12\cos(6x+7y)}{14\cos(6x+7y) - 5x} \quad \checkmark$$

(3) a spherical balloon's radius is decreasing at a rate of 1.2 cm/sec

a) How fast is the volume decreasing when $r = 10\text{cm}$

$V = \frac{4}{3}\pi r^3$ V: volume of sphere t: time

r: radius of sphere.

$\frac{dV}{dt}$ = How fast volume changes $\left.\frac{dV}{dt}\right|_{r=10} = ??$

$$\frac{dr}{dt} = -1.2$$

$$\frac{dV}{dt} = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$$

$$= \frac{4}{3}\pi \frac{d}{dt}\left[r^3\right]$$

$$= \frac{4}{3}\pi \cdot 3(r)^2 \cdot \frac{dr}{dt}$$

$$= \frac{4}{3}\pi \cdot 3(10)^2 \cdot (-1.2) \quad cm^3/sec$$

b) How fast is the surface area decreasing when $r = 10cm$

$A = 4\pi r^2$     A: surface area    r: radius    t: time.

c number

$$\left.\frac{dA}{dt}\right|_{r=10} = ??$$

$$\frac{d}{dx}\left[c f(x)\right] = c\frac{d}{dx}\left[f(x)\right]$$

$$\frac{dA}{dt} = \frac{d}{dt}\left[4\pi r^2\right]$$

$$= 4\pi \frac{d}{dt}\left[r^2\right]$$

$$= 4\pi \cdot 2(r)^1 \cdot \frac{dr}{dt}$$

$$= 4\pi \cdot 2(10) \cdot (-1.2)$$

$$= -8\pi \cdot 12$$

$$= -96\pi \quad cm^2/sec$$

The surface area is decreasing at a rate of $96\pi$ $cm^2/sec$.

④ $y = x^4 + 2x^3 - 12x^2 + 12x - 36$

find inflection points.

$$y' = 4x^3 + 6x^2 - 24x + 12$$

$$y'' = 12x^2 + 12x - 24$$

$$= 12(x^2 + x - 2)$$

$$= 12(x+2)(x-1)$$

$$y(-2) = (-2)^4 + 2(-2)^3 - 12(-2)^2 + 12(-2) - 36$$

$$= 16 - 16 - 48 - 24 - 36$$

$$= -108$$

$$y(1) = (1)^4 + 2(1)^3 - 12(1)^2 + 12(1) - 36$$

$$= 1 + 2 - 12 + 12 - 36$$

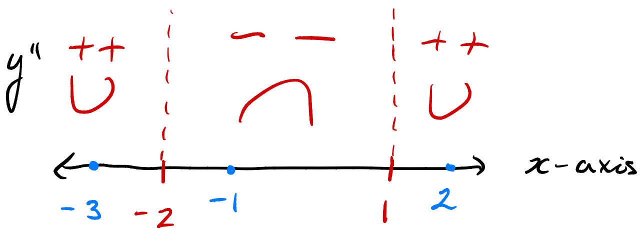
$$= -33$$

$y'' = 0$

$x = -2, +1$

$y''$ DNE

never happens




$$y''(-1) = \overset{+}{12}(\overset{+}{-1+2})(\overset{-}{-1-1}) < 0$$

concave down $(-2, 1)$

$$y''(-3) = \underset{+}{12}(\underset{-}{-3+2})(\underset{-}{-3-1}) > 0$$

concave up $(-\infty, -2)$

$$y''(2) = \underset{+}{12}(\underset{+}{2+2})(\underset{+}{2-1}) > 0$$

concave up on $(1, \infty)$

Inflection points : $(-2, -108)$ and $(1, -33)$ ✓

Review for exam #2

**Chain rule** $\quad y = f(g(x))$

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**Implicit differentiation**

$y' = \frac{dy}{dx} = \frac{d}{dx}[y]$ where $y$ is a function of $x$.

① $6\tan(x/y) = 13x$ find $dy/dx$

$$\frac{d}{dx}\left[6\tan(x/y)\right] = \frac{d}{dx}[13x]$$

$$6\sec^2\left(\frac{x}{y}\right)\cdot\frac{d}{dx}\left[\frac{x}{y}\right] = 13$$

$$6\sec^2\left(\frac{x}{y}\right)\cdot\frac{1\cdot y - xy'}{y^2} = 13$$

$\sec x = \frac{1}{\cos x}$

$$\frac{y - xy'}{y^2} = \frac{13}{6}\cos^2(x/y)$$

$$y - xy' = \frac{13}{6}y^2\cos^2(x/y)$$

$$xy' = y - \frac{13}{6}y^2\cos^2(x/y)$$

$$y' = y/x - \frac{13}{6}\frac{y^2}{x}\cos^2(x/y) \quad ✓$$

(2) $2\sin(6x+7y) = 5xy$ find $dy/dx$

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$$y' = \frac{5y - 12\cos(6x+7y)}{14\cos(6x+7y) - 5x}$$

(3) A spherical balloon's radius is decreasing at a rate of 1.2 cm/sec.

a) How fast is the volume decreasing when $r = 10$ cm.

$V = \frac{4}{3}\pi r^3$ V: volume r: radius t: time

$\left.\frac{dV}{dt}\right|_{r=10} = ??$ $\frac{dr}{dt} = -1.2$

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \frac{d}{dt}[r^3]$$

$$= \frac{4}{3}\pi \cdot 3(r)^2 \cdot \frac{dr}{dt}$$

$$\left.\frac{dV}{dt}\right|_{r=10} = \frac{4}{3}\pi \cdot 3(10)^2 \cdot (-1.2)$$

$$= 4\pi \cdot 10 \cdot (-12)$$

$$= -480\pi \text{ cm}^3/\text{sec}$$

The volume is decreasing at a rate of $480\pi$ cm$^3$/sec. ✓

b) How fast is the surface area decreasing when $r = 10$ cm.

$A = 4\pi r^2$ A: surface area, r: radius, t: time.

$\left.\frac{dA}{dt}\right|_{r=10} = ??$ $\frac{dr}{dt} = -1.2$

$$\frac{dA}{dt} = \frac{d}{dt}[4\pi r^2]$$

$$= 4\pi \frac{d}{dt}[r^2]$$

$$= 4\pi \cdot 2(r)^1 \cdot \frac{dr}{dt}$$

$$\left.\frac{dA}{dt}\right|_{r=10} = 4\pi \cdot 2 \cdot 10 \cdot (-1.2)$$

$$= -96\pi \text{ cm}^2/\text{sec}$$

So rate at which SA is decreasing is $96\pi$ cm$^2$/sec.

(4) $y = x^4 + 2x^3 - 12x^2 + 12x - 36$ find inflection points

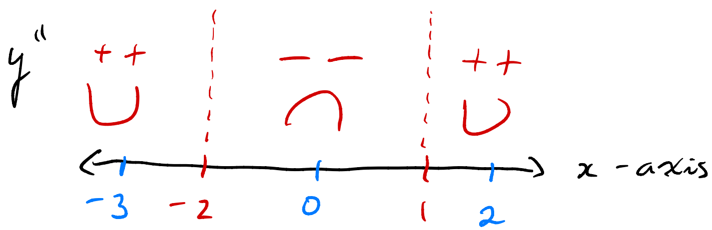
$y' = 4x^3 + 6x^2 - 24x + 12$

$y'' = 12x^2 + 12x - 24$

$= 12(x+2)(x-1)$

$y'' = 0$ or $y''$ DNE never happens

$x = -2, 1$




$y''(-3) = 12(-3+2)(-3-1) > 0$
$\quad\quad\quad\; + \quad\;\; - \quad\quad\;\; -$

Concave up on $(-\infty, -2)$

$y''(0) = 12(0+2)(0-1) < 0$
$\quad\quad\; + \quad\; + \quad\quad -$

concave down on $(-2, 1)$

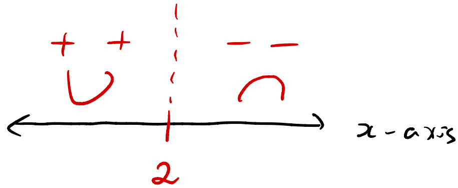
$y''(2) = 12(2+2)(2-1) > 0$

Concave up $(1, \infty)$

$(-2, -108)$ $(1, -33)$ are the inflection pts.

5) $f(x)$ is a polynomial

a) $f'(-1) = 0$ $f'(3) = 0$




b) $f''(2) = 0$

c) $f''(x) > 0$ on $(-\infty, 2)$

$f''(x) < 0$ on $(2, \infty)$

find rel ext. and infl.

We have critical pts at $x = -1, 3$

$f''(-1) > 0$, 2nd deriv. test $\Rightarrow$ $x = -1$ rel. min

$f''(3) < 0$, 2nd deriv test $\Rightarrow$ $x = 3$ is rel max ✓