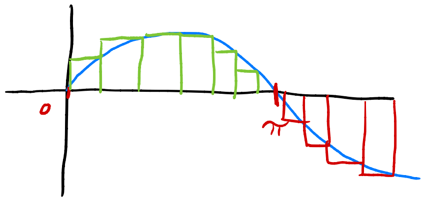
HW 28 #10 $y = -2\sin(x+\pi)$. Estimate the signed area under the curve y on $[0, 2\pi]$ w/ 40 rect.

Right Riemann Sums

$$A \approx \sum_{i=1}^{40} y(x_i)\Delta x$$



$$\Delta x = \text{width of rect.}$$

$$= \frac{2\pi - 0}{40} = \frac{\pi}{20}$$

$$x_i = 0 + i\Delta x$$

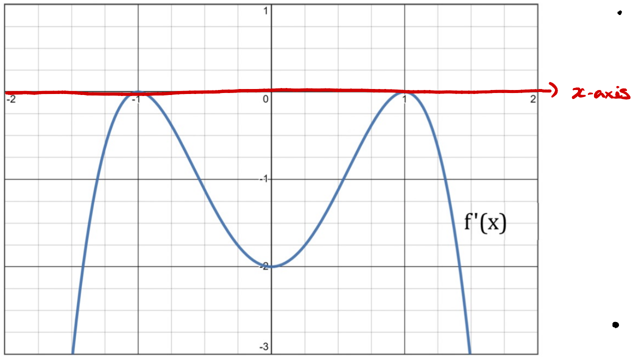
$$A \approx \sum_{i=1}^{40} y(i\Delta x)\,\Delta x = \sum_{i=1}^{40} -\sin\left(i\frac{\pi}{20} + \pi\right)\cdot\frac{\pi}{20}$$

Left Riemann Sum

$$\sum_{i=0}^{39} y(x_i)\Delta x = \sum_{i=0}^{39} -\sin\left(i\frac{\pi}{20} + \pi\right)\cdot\frac{\pi}{20}$$

Review for Exam #3 : Lesson 20–28

(1)



- find the relative extrema of f.

critical pts : $x = \pm 1$

no rel. ext.

- Inc / Dec

Inc : nowhere

Dec : $(-\infty, \infty)$

- Concavity

Concave up : $(-\infty, -1) \cup (0, 1)$

$f'' > 0$ or f' to be increasing

Concave down : $(-1, 0) \cup (1, \infty)$

$f'' < 0$ or f' is decreasing.

Limits at Infinity

(2) $\lim_{x \to \infty} \frac{x^3 + 1}{x - 2} = +\infty$

$\lim_{x \to -\infty} \frac{x^3 + 1}{x - 2} = +\infty$

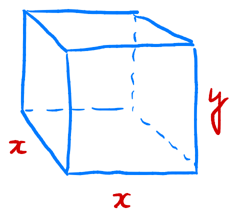
$\sim \frac{x^3}{x} = x^2$

(3) $\lim_{x \to \infty} \frac{x^2 + x - 1}{3x - x^2} = -1$

$\lim_{x \to -\infty} \frac{x - 6}{x^5 + 100x} = 0$

$\sim \frac{1}{x^4}$

⑤ A rectangular box has square base.
The sum of the height of the box
a perimeter of the base is 12 in.
What is max volume of the box.



Constraint: $12 = 4x + y$; $x, y > 0$

Obj: $V = x^2 y$

$y = 12 - 4x$

$0 < 12 - 4x$

$4x < 12$

$x < 3$

New Obj: $V = x^2(12 - 4x)$
$= 12x^2 - 4x^3$

New Const: $0 < x < 3$

Find the abs. max of $V = 12x^2 - 4x^3$ on $(0, 3)$.

$V' = 24x - 12x^2 = 12x(2 - x)$

$V' = 0$ $\qquad$ V' DNE doesn't happen.

$0 = 12x(2 - x)$

$\cancel{x = 0}$ $\qquad$ $x = 2$

ignore since 0 not in $(0, 3)$.

$V'' = 24 - 24x$

$V''(2) = 24 - 24(2) < 0$

we know $x = 2$ is a rel. max.

So $x = 2$ is the x-value of the abs. max on $(0, 3)$.

$V = x^2 y = x^2(12 - 4x)$

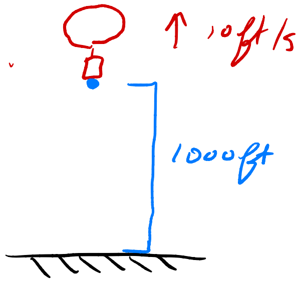
$V(2) = 2^2(12 - 4(2))$

$= 4(12 - 8)$

$= 4 \cdot 4$

$= 16 \text{ in}^3$

(6) A hot air balloon rise vertically at a rate of 10 ft / sec. Use $a(t) = -32 \text{ ft/sec}^2$ as accelerations.



How long does it take for ball to reach the ground.

$$v(t) = \int a(t)\, dt \qquad v(t) = \int -32\, dt = -32t + C_1$$

$$s(t) = \int v(t)\, dt \qquad \underset{10}{\overset{\shortparallel}{v(0)}} = -32(0) + C_1$$

$$C_1 = 10$$

$$s(t) = \int -32t + 10\, dt$$

$$= -16t^2 + 10t + C_2$$

$$\underset{1000}{\overset{\shortparallel}{s(0)}} = -16(0)^2 + 10(0) + C_2$$

$$C_2 = 1000$$

$$s(t) = -16t^2 + 10t + 1000 = 0$$

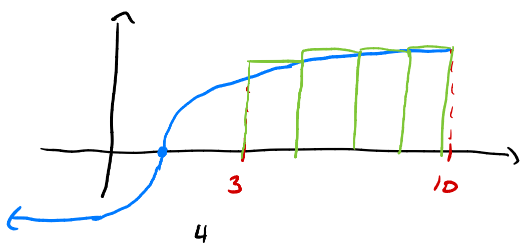
Solve for t using the quadratic formula.

HW 28 #7 | Estimate the signed area under the curve $y = \sqrt[3]{x-1}$ on $[3, 10]$ using 4 rect.

Right Riemann Sums

width of our rectangles

$$\Delta x = \frac{10-3}{4} = \frac{7}{4}$$



$$x_i = 3 + i\Delta x$$
$$= 3 + i \cdot 7/4$$

$$\sum_{i=1}^{4} y(x_i)\,\Delta x$$

$$\sum_{i=1}^{4} \sqrt[3]{3 + i \cdot \tfrac{7}{4} - 1}\left(\tfrac{7}{4}\right)$$

$$= \sqrt[3]{3 + 1 \cdot \tfrac{7}{4} - 1}\left(\tfrac{7}{4}\right)$$
$$+ \sqrt[3]{3 + 2 \cdot \tfrac{7}{4} - 1}\left(\tfrac{7}{4}\right)$$
$$+ \sqrt[3]{3 + 3 \cdot \tfrac{7}{4} - 1}\left(\tfrac{7}{4}\right)$$
$$+ \sqrt[3]{3 + 4 \cdot \tfrac{7}{4} - 1}\left(\tfrac{7}{4}\right)$$

left Riemann Sums

$$\sum_{i=0}^{4-1} y(x_i)\,\Delta x = \sum_{i=0}^{3} \sqrt[3]{3 + i \cdot \tfrac{7}{4} - 1}\left(\tfrac{7}{4}\right)$$

Review for Exam #3 : Lessons 20-28

(1) $\lim\limits_{x\to\infty} \dfrac{x^3+1}{x-2} \sim \dfrac{x^3}{x} = x^2$ $\qquad$ $\lim\limits_{x\to-\infty} \dfrac{x^3+1}{x-2}$

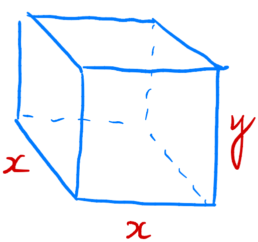
$= +\infty$ $\qquad$ $= +\infty$

$\lim\limits_{x\to\infty} \dfrac{x^2+x-1}{3x-x^2}$ $\qquad$ $\lim\limits_{x\to-\infty} \dfrac{x-100}{x^5+1} \sim \dfrac{x}{x^5} = \dfrac{1}{x^4}$

$\sim \dfrac{\cancel{x^2}}{\cancel{-x^2}} = -1$ $\qquad$ $= 0$

$= -1$

(2) A rectangular box has a square base. & the sum of the perimeter of the base and the height of box is 12 in. What is the max. volume of the box?



Const: $12 = 4x + y$; $x, y > 0$

Obj: $V = x^2 y$

$y = 12 - 4x > 0$ $\qquad$ New Obj: $V = x^2(12-4x)$

$4x < 12$ $\qquad$ $= 12x^2 - 4x^3$

$x < 3$ $\qquad$ New Const: $0 < x < 3$

Find the abs. max of $V = 12x^2 - 4x^3$ on $(0,3)$.

$V' = 24x - 12x^2 = 12x(2-x)$

$V' = 0$ $\qquad$ V' DNE doesn't have.

$0 = 12x(2-x)$

$\cancel{x=0}$ $\qquad x = 2$

ignore since 0 is not in $(0,3)$!

$V'' = 24 - 24x$

$V''(2) = 24 - 24(2) < 0$

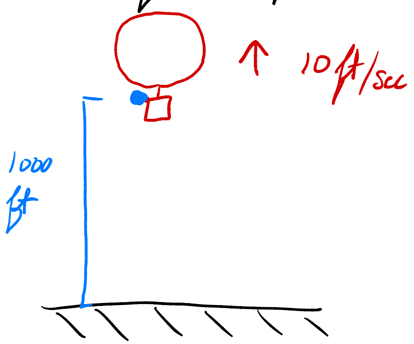
so $x=2$ is a rel. max.

So $x=2$ is an abs. max on $(0,3)$

$V = x^2 y = x^2(12-4x)$

$V(2) = 2^2(12 - 4(2)) = 16 \text{ in}^3$.

③ A hot air balloon rising vertically at a rate of 10 ft/sec. (Acceleration due to grav. $a(t) = -32\ \text{ft}/\text{sec}^2$



How long does it take to hit the ground?

phys facts $\left\{ \begin{array}{l} v(t) = \int a(t)\,dt \\ s(t) = \int v(t)\,dt \end{array} \right.$,

$$v(t) = \int -32\,dt = -32t + c_1$$

$$10 = v(0) = -32(0) + c_1$$

$$c_1 = 10$$

$$v(t) = -32t + 10.$$

$$s(t) = \int -32t + 10\,dt = -16t^2 + 10t + c_2$$

$$1000 = s(0) = -16(0)^2 + 10(0) + c_2$$

$$c_2 = 1000.$$

$$s(t) = -16t^2 + 10t + 1000 = 0$$

$$t = \frac{-10 \pm \sqrt{10^2 - 4(-16)(1000)}}{2(-16)}$$

$t \approx \cancel{-7.6}$ sec or $\underline{8.2 \text{ sec}}$

b) What is the velocity of the ball when it hits the ground?

$$v(t) = -32t + 10$$

$$v(8.2) = -32(8.2) + 10$$

$$= -252.4 \text{ ft/sec}$$