# Review for Final I

Alt. Exam 3 | Colony of bacteria grows at a rate proportional to the square of time in hours w/ a constant coeff of 1200.

Starts w/ 400 bacteria

How many bacteria in 3.5 hrs.

$P(t)$ = pop. of Bact. $\qquad$ $t$ = time in hours

$$\frac{dP}{dt} = \text{growth rate} = 1200t^2 \qquad P(0) = 400$$

$$P(3.5) = ??$$

$$P(t) = \int 1200t^2\,dt$$

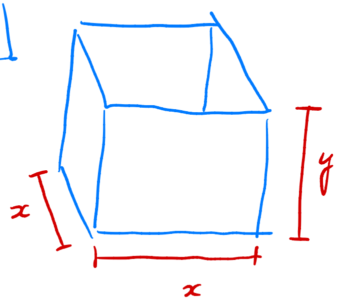
$$= 400t^3 + C$$

$$400 = P(0) = 400(0)^3 + C$$

$$C = 400$$

$$P(3.5) = 400(3.5)^3 + 400 = 17{,}550 \text{ Bacteria}$$

Exam 3




Sides + bot are wood

bot also has carpet

square base

wood = \$3 / $ft^2$ carpet = \$1 / $ft^2$

Cat requires 18 $ft^3$

Minimize cost of box.

obj: $C = 3\left(4(xy) + x^2\right) + 1\left(x^2\right)$

$= 12xy + 3x^2 + x^2$

$= 12xy + 4x^2$

const.: $18 = x^2 y$ $\quad x, y > 0$

$y = \dfrac{18}{x^2}$

new obj: $C = 12x\left(\dfrac{18}{x^2}\right) + 4x^2$

$= \dfrac{216}{x} + 4x^2$

$= \dfrac{216 + 4x^3}{x}$

new constraint: $0 < x < +\infty$

$0 < y = \frac{18}{x^2}$ always true

We want to find the abs. min of

$C = \frac{216 + 4x^3}{x}$ on $(0, \infty)$

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$$C' = \frac{12x^2 \cdot x - (216 + 4x^3)}{x^2}$$

$$= \frac{8x^3 - 216}{x^2}$$

Critical numbers: $C' = 0$ $\quad$ $C'$ DNE

$\cancel{x \neq 0}$

not in $(0, \infty)$

$C' = 0$

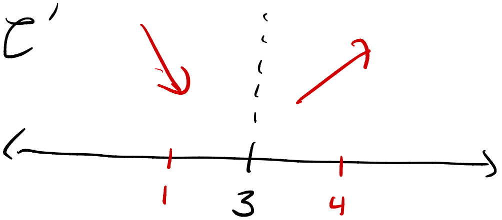
$8x^3 - 216 = 0$

$8x^3 = 216$

$x^3 = 27$

$x = 3$ is in $(0, \infty)$

$C'$




$x = 3$ is a abs. min of $C$.

$C(3) = \frac{216 + 4(3)^3}{3} = \$108$ ✓

Practice after Exam 3 #8

$$\int_1^3 \ln(x^2+3)\,dx$$

Approximate using $n=4$ trapezoids.

$$f(x) = \ln(x^2+3)$$

$$\Delta x = \frac{3-1}{4} = \frac{1}{2}$$

$$x_i = 1 + i\left(\tfrac{1}{2}\right)$$

$$\int_1^3 \ln(x^2+3)\,dx \approx T_4 = \frac{\Delta x}{2}\Big[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\Big]$$

$$x_0 = 1 + 0\left(\tfrac{1}{2}\right) = 1$$

$$x_1 = 1 + 1\left(\tfrac{1}{2}\right) = 3/2$$

$$x_2 = 1 + 2\left(\tfrac{1}{2}\right) = 2$$

$$x_3 = 1 + 3\left(\tfrac{1}{2}\right) = 5/2$$

$$x_4 = 1 + 4\left(\tfrac{1}{2}\right) = 3$$

$$= \frac{1/2}{2}\Big[\ln\left((1)^2+3\right) + 2\ln\left(\left(\tfrac{3}{2}\right)^2+3\right) + 2\ln\left(2^2+3\right) + 2\ln\left(\left(\tfrac{5}{2}\right)^2+3\right) + \ln\left(3^2+3\right)\Big]$$

# Review for Final I

## Practice problems after Exam 3 #8

$$\int_1^3 \ln(x^2+3)\,dx$$

Approx. w/ $n=4$ trapezoid.

$$f(x) = \ln(x^2+3)$$

$$\int_1^3 \ln(x^2+3)\,dx \approx T_4$$

$$= \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$$

$$\Delta x = \frac{3-1}{4} = \frac{1}{2}$$

$$x_i = 1 + i\left(\tfrac{1}{2}\right)$$

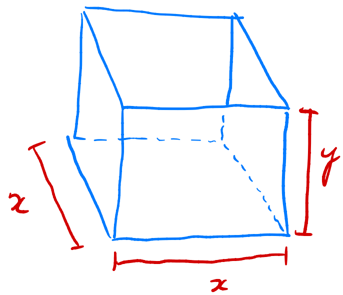
$x_0 = 1$ $\qquad$ $x_3 = 5/2$

$x_1 = 3/2$ $\qquad$ $x_4 = 3$

$x_2 = 2$

$$T_4 = \frac{1/2}{2}\left[\ln(1^2+3) + 2\ln\left(\left(\tfrac{3}{2}\right)^2+3\right) + 2\ln(2^2+3) + 2\ln\left(\left(\tfrac{5}{2}\right)^2+3\right) + \ln(3^2+3)\right]$$

Exam #3




wood on sides + bot.

carpet on bot.

square base

wood = \$3 / $ft^2$ carpet = \$1 / $ft^2$

Cat needs $18 ft^3$

Minimize cost of box.

obj: $C = 3(4xy + x^2) + 1(x^2)$

$= 12xy + 4x^2$

Const: $18 = x^2 y \qquad x, y > 0$

$y = \dfrac{18}{x^2}$

new obj: $C = 12x\left(\dfrac{18}{x^2}\right) + 4x^2$

$= \dfrac{216}{x} + 4x^2$

$= \dfrac{216 + 4x^3}{x}$

new const: $0 < x < \infty$

$0 < y = \dfrac{18}{x^2}$

We want to find the abs. min of $C = \dfrac{216 + 4x^3}{x}$ on $(0, \infty)$.

$$C' = \frac{12x^2 \cdot x - (216 + 4x^3) \cdot 1}{x^2}$$

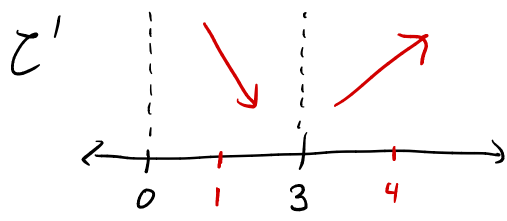
$$= \frac{8x^3 - 216}{x^2}$$

Critical numbers: $C' = 0$ or $C'$ DNE

$\cancel{x = 0}$ not in $(0, \infty)$

$C' = 0$

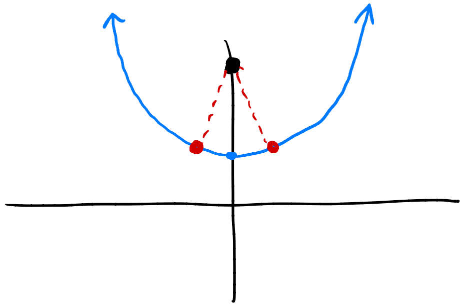
$$8x^3 - 216 = 0$$
$$8x^3 = 216$$
$$x^3 = 27$$
$$x = 3$$



$x = 3$ is an abs. min of $C$ on $(0, \infty)$.

$$C(3) = \frac{216 + 4(3)^3}{(3)} = \$108$$

Lesson 25 | Find pts on $y = x^2 + 1$ closest to the point $(0, 5)$.



$(0, 5)$
$\parallel$
distance b/w two points $(x, y)$ and $(x_0, y_0)$ is

$$d = \sqrt{(x - x_0)^2 + (y - y_0)^2}$$

$$D = d^2 = (x - x_0)^2 + (y - y_0)^2$$

Obj: $D = (x - 0)^2 + (y - 5)^2$

$= x^2 + (y - 5)^2$

Const: $y = x^2 + 1$