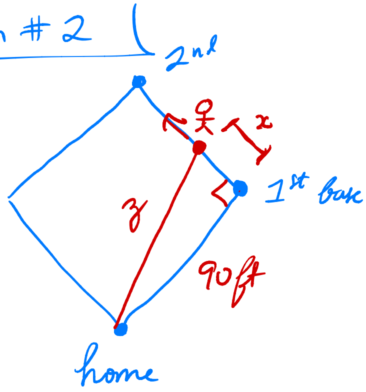
Review for Final II

Exam # 2



Square w/ side lengths 90 ft

person runs at 11 ft/sec

find the rate at which the distance between the player and home plate changes when they half way between 1^{st} and 2^{nd}.

let t be time in sec.

$$\frac{d}{dt}\left[x^2 + 90^2 = z^2 \right] \qquad \frac{dx}{dt} = 11 \text{ ft/sec} \qquad \frac{dz}{dt} = ??$$

$$2x \cdot \frac{dx}{dt} + 0 = 2z\,\frac{dz}{dt}$$

$$\frac{11x}{z} = \frac{22x}{2z} = \frac{dz}{dt}$$

when player is half way between 1^{st} and 2^{nd}, then $x = 45$ (half of 90)

$$45^2 + 90^2 = z^2$$

$$z = \sqrt{10125} = \sqrt{5^3 \cdot 3^4} = 5 \cdot 3^2 \sqrt{5}$$

$$10125 = 5 \cdot 2025, \quad 2025 = 5 \cdot 405, \quad 405 = 5 \cdot 81, \quad 81 = 3^4$$

$$\frac{dy}{dt} = \frac{11x}{y} = \frac{11(45)}{5\cdot 3^2\sqrt{5}} = \frac{11}{\sqrt{5}} \text{ ft/sec}$$

After Exam 3 | R.o.C of a pop. of bacteria is $P'(t) = 2\sqrt{t}\,(10t+3)$, where t is in hours. What is the increase in the bacteria pop. between $t=4$ and $t=9$.

↳ means Net Change from $t=4$ to $t=9$

i.e.. definite integral from $t=4$ to $t=9$

$$\int_4^9 P'(t)\,dt = \int_4^9 2\underbrace{\sqrt{t}}_{t^{1/2}}\,(10t+3)\,dt$$

$$= 2\int_4^9 10t^{3/2} + 3t^{1/2}\,dt$$

$$\overset{FTC}{=} 2\left(10\cdot\tfrac{2}{5}t^{5/2} + 3\cdot\tfrac{2}{3}\cdot t^{3/2}\right)\Big|_4^9$$

$$= 2\left(4t^{5/2} + 2t^{3/2}\right)\Big|_4^9$$

$$= 2\left[4\cdot 9^{5/2} + 2\cdot 9^{3/2} - \left(4\cdot 4^{5/2} + 2\cdot 4^{3/2}\right)\right]$$

$$= 2\left[4\cdot 3^5 + 2\cdot 3^3 - 2^7 - 2^4\right]$$

$$= 1764 \text{ bacteria}$$

$v_A = t + 2 \qquad v_B = 2t$

displacement from time 0 to t_0 $= \int_0^{t_0} v \, dt$

$$\int_0^{t_0} t + 2 \, dt = \left(\frac{1}{2}t^2 + 2t\right)\Big|_0^{t_0}$$

$$= \frac{1}{2}t_0^2 + 2t_0$$

$$\int_0^{t_0} 2t \, dt = t^2\Big|_0^{t_0} = t_0^2$$

$$\frac{1}{2}t_0^2 + 2t_0 = t_0^2$$

$$0 = \frac{1}{2}t_0^2 - 2t_0$$

$$= t_0^2 - 4t_0$$

$$= t_0(t_0 - 4)$$

$$t_0 = 0 \qquad t_0 = 4$$