Review for Final III

Exam 2 $\quad 2xy = y^3 + 3$ $\qquad \underset{\shortparallel}{x} \; \underset{\shortparallel}{y}$

Use implicit diff. to find dy/dx at $(2, 1)$.

y is a function of x

$$\frac{d}{dx}\left[2xy\right] = \frac{d}{dx}\left[y^3 + 3\right]$$

$$2y + (2x)\frac{dy}{dx} = 3(y)^2 \cdot \frac{dy}{dx} + 0$$

$$2x\frac{dy}{dx} - 3y^2\frac{dy}{dx} = -2y$$

$$\frac{dy}{dx}\left(2x - 3y^2\right) = -2y$$

$$\frac{dy}{dx} = \frac{-2y}{2x - 3y^2}$$

$$= \frac{-2(1)}{2(2) - 3(1)^2}$$

$$= \frac{-2}{1}$$

$$= -2$$

Exam 2 $f(x)$ is a polynomial

$f'(1) = 0$ $f'(4) = 0$

$f''(2.5) = 0$ $f''(x) < 0$ for $x < 2.5$

$f''(x) > 0$ for $x > 2.5$

I. $(1, f(1))$ is an inflection pt of $f(x)$.

(x, y) is an inflection point if

- $f''(x) = 0$ or DNE
- switch concavity at (x, y).

$f''(1) < 0$

fail

II $(2.5, f(2.5))$ is an inflection point.

$f''(2.5) = 0$

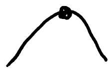
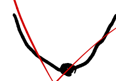
concave up $\Leftrightarrow$ $f'' > 0$

concave down $\Leftrightarrow$ $f'' < 0$

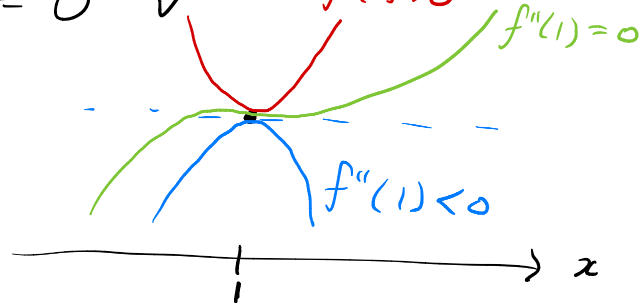
we do switch concavity

True

III $f(x)$ has a relative maximum at $x = 1$

- critical point 
- switch from inc → dec or ~~dec → inc.~~ 

$f'(1) = 0$ ✓



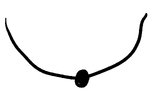
We know: $f''(x) < 0$ for $x < 2.5$

$\Longrightarrow f''(1) < 0$

True.

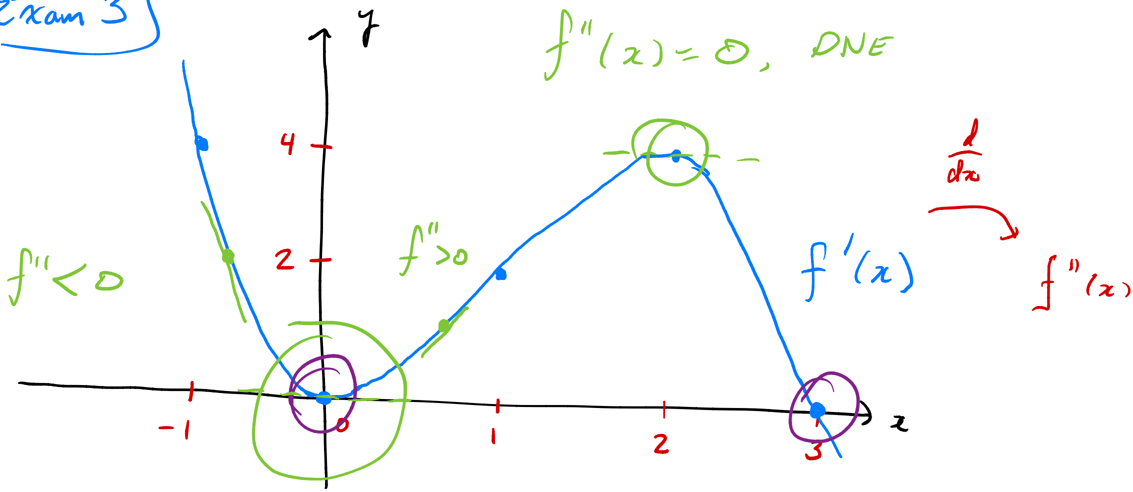
IV $f(x)$ has a rel. min at $x = 4$

- critical number $f' = 0$, DNE
- dec → inc

 ⟶ $f'' > 0$

$f'(4) = 0$ ✓, $f''(4) > 0$ ✓ True.

Exam 3



Critical points: $x = 0$ (neither) $\quad x = 3$ (rel max)

rel min, $\quad - \searrow \quad + \nearrow$

rel max $\quad + \nearrow \quad - \searrow$

neither $\quad + \nearrow \quad + \nearrow \qquad - \searrow \quad - \searrow$

Inflection $\quad x = 0 \qquad x = 2$

f'' to switch signs

Both inflection points.

after Exam 3 #55

Car travelling 60 mph $\rightsquigarrow$ $v(0) = 60$ mph

acceleration after breaks $a(t) = -(t-3)^2$ mph/s

$|v(3)| = ??$

velocity

$$v(t) = \int a(t)\,dt$$

$$= \int -(t-3)^2\,dt$$

$$= \int -(t^2 - 6t + 9)\,dt$$

$$= \int -t^2 + 6t - 9\,dt$$

$$v(t) = -\frac{1}{3}t^3 + 3t^2 - 9t + c$$

$$60 = -\frac{1}{3}(0)^3 + 3(0)^2 + 9(0) + c$$

$$60 = c$$

$$v(t) = -\frac{1}{3}t^3 + 3t^2 - 9t + 60$$

$$v(3) = -\frac{1}{3}(3)^3 + 3(3)^2 - 9(3) + 60$$

$$= -3^2 + 3^3 - 3^3 + 60$$

$= 51$ mph

$|v(5)| = 51$ mph.

After Exam 3 # 28

half life of ^{14}C = 5715 years

mummy has 70% of the amount of ^{14}C of a living human.

how old is the mummy?

half life

$$k = \frac{-\ln 2}{\text{half life}}$$

$$\frac{1}{2} = e^{kt}$$

P = ammount of ^{14}C

t = time in years

$$P(t) = ce^{kt} \qquad k = \frac{-\ln 2}{5715}$$

If a living human has c grams of ^{14}C

then the mummy has ce^{kt} grams of ^{14}C

$$.70 = \frac{\text{ammount of } ^{14}C \text{ in mummy}}{\text{ammount of } ^{14}C \text{ in a living human.}}$$

$$= \frac{ce^{kt}}{c}$$

$$.70 = e^{kt}$$

$$\ln(.7) = kt$$

$$t = \frac{\ln(.7)}{k} = \frac{\ln(.7)}{-\frac{\ln 2}{5715}}$$

$$\approx 2941 \text{ years.}$$

Practice Exam 2 #2

$$f(x) = \frac{2(3-x^2)}{\sqrt{3x^2+1}} \qquad f'(1) = ??$$

$f'(x)$

$$= \frac{\frac{d}{dx}\left[2(3-x^2)\right]\sqrt{3x^2+1} - 2(3-x^2)\frac{d}{dx}\left[\sqrt{3x^2+1}\right]}{\left(\sqrt{3x^2+1}\right)^2}$$

$$= \frac{2(0-2x)\sqrt{3x^2+1} - 2(3-x^2)\cdot\frac{1}{2}(3x^2+1)^{-\frac{1}{2}}(6x)}{3x^2+1}$$

$f(1)$

$$= \frac{2(-2(1))\sqrt{3(1)^2+1} - 2(3-(1)^2)\cdot\frac{1}{2}(3(1)^2+1)^{-\frac{1}{2}}\cdot 6}{3(1)^2+1}$$

$$= \frac{-8 + {}^{-}6}{4}$$

$$= \frac{-14}{4} = \frac{-7}{2}$$

After Exam 3 # 9

$$\frac{dy}{dt} = -3y \qquad y(1) = 5, \quad y(10) = ??$$

$$\Downarrow$$

$$y = ce^{-3t}$$

$$x^a x^b = x^{a+b}$$

$$\frac{1}{e^{-3}} = e^3$$

$$(e^3)5 = y(1) = ce^{-3}(e^3) = ce^{-3+3}$$

$$= c\cancel{e^0}$$

$$c = 5e^3$$

$$y(t) = 5e^3e^{-3t} = 5e^{-3t+3}$$

$$y(10) = 5e^{-30+3} = 5e^{-27}$$

Practice Exam 1 # 5

Pop. of a city is $P(t) = 10 + \dfrac{50t}{2t^2+9}$

t time in years.

What is R. o. C of pop. at $t = 2$?

derivative P(t)

$$\left.\frac{dP}{dt}\right|_{t=2}$$

$$\frac{d}{dt}\left[10 + \frac{50t}{2t^2+9}\right]$$

$$0 + \frac{50(2t^2+9) - 50t(4t+0)}{(2t^2+9)^2}$$

$$\left.\frac{dP}{dt}\right|_{t=2} = \frac{50(2(2)^2+9) - 50(2)(4(2)+0)}{(2(2)^2+9)^2}$$

$$= \frac{50\cdot 17 - 50\cdot 16}{17^2}$$

$$= \frac{50}{17^2} \approx .173 \text{ people/year}$$