Exam # 1 Review

1. $f(x) = \begin{cases} \cos x & x \le 0 \\ x+1 & 0 < x \le 1 \\ -x & x > 1 \end{cases}$

find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$.

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \cos x = \cos(0) = 1$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x+1) = 0+1 = 1$$

$$\lim_{x\to 0} f(x) = 1$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x+1) = 1+1 = 2$$

$$\neq$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (-x) = -(1) = -1$$

$\lim_{x\to 1} f(x)$ DNE.

2. $g(x) = \dfrac{x^2+3x}{x^2-x}$ find / classify the discontinuities

$$g(x) = \frac{x(x+3)}{x(x-1)}$$ discont @ $x=0$ and $x=1$

$\underline{x=1}$ $g(1) = \dfrac{1\cdot 4}{0}$ "$\dfrac{\text{nonzero}}{0}$"

So this is an asymptote

x	.99	.999	1	1.001	1.01
$f(x)$	~	~		~	~

$\longrightarrow -\infty$ $\quad$ $+\infty \longleftarrow$

$\boxed{x=0}$ $\quad \lim_{x\to 0} g(x) = \lim_{x\to 0} \frac{\cancel{x}(x+3)}{\cancel{x}(x-1)} = \lim_{x\to 0} \frac{x+3}{x-1} = \frac{0+3}{0-1} = -3$

a number

Since g is not defined at $x=0$, but $\lim_{x\to 0} g$ exists (and is not $\pm\infty$) then

$x=0$ is a hole of g.

③ find the tangent line to $f(x) = 3e^x \tan x$ at $x = \frac{\pi}{4}$

point-slope form $\quad y - y_1 = m(x - x_1)$

m = slope at $x = \pi/4$

(x_1, y_1) = point on the line

$x_1 = \pi/4 \qquad y_1 = f(\pi/4) = 3e^{\pi/4} \qquad \sec x = \frac{1}{\cos x}$

$$f'(x) = \frac{d}{dx}\left[3e^x \tan x\right] = 3e^x \tan x + 3e^x \sec^2 x$$

$$m = f'\left(\tfrac{\pi}{4}\right) = 3e^{\pi/4} + 3e^{\pi/4} \cdot \left(\frac{1}{\cos \pi/4}\right)^2$$

$$= 3e^{\pi/4}\left(1 + \left(\frac{1}{\sqrt{2}/2}\right)^2\right)$$

$$= 3e^{\pi/4}\left(1 + 2\right)$$

$$= 9e^{\pi/4}$$

$$y - 3e^{\pi/4} = 9e^{\pi/4}(x - \pi/4)$$

$$\boxed{y = 9e^{\pi/4}x - \frac{9}{4}\pi e^{\pi/4} + 3e^{\pi/4}}$$

4) Suppose a particle is moving in a straight line with position given by

$$S(t) = \left(\frac{t^3+1}{\cos t}\right)^5$$

in meters. What is the velocity of the particle?

$$v(t) = \frac{d}{dt}\left[\left(\frac{t^3+1}{\cos t}\right)^5\right] = g'(h(x)) \cdot h'(x)$$

outside: $g(u) = u^5$

$g'(u) = 5u^4$

inside $h(x) = \dfrac{t^3+1}{\cos t}$

$$h'(x) = \frac{d}{dx}\left[\frac{t^3+1}{\cos t}\right]$$

$$= \frac{(3t^2)\cos t - (t^3+1)(-\sin t)}{\cos^2 t}$$

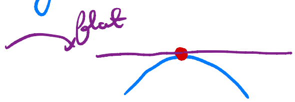
$$= \frac{3t^2\cos t + (t^3+1)\sin t}{\cos^2 t}$$

$$v(t) = 5\left(\frac{t^3+1}{\cos t}\right)^4 \cdot \frac{3t^2\cos t + (t^3+1)\sin t}{\cos^2 t}$$

↑ velocity of particle.

(5) $f(x) = x^3 - 3x$. For what x values do we have horizontal tangent lines?

horizontal tan line



is when does $f'(x) = 0$?

ie. $f'(x) = 0$ solve for x.

$$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x+1)(x-1)$$

$$f'(x) = 0$$

$$3(x+1)(x-1) = 0$$

$$\boxed{x = 1 \quad \text{or} \quad x = -1}$$

Exam #1 Review

to be cont. need
1) $f(c)$ defined
2) $\lim_{x \to c} f(x)$ exist
3) $f(c) = \lim_{x \to c} f(x)$

① $$f(x) = \begin{cases} \cos x & x \leq 0 \\ x+1 & 0 < x \leq 1 \\ -x & x > 1 \end{cases}$$

find and classify the discontinuities.

$x = 0$

1) $f(0) = \cos(0) = 1$

2) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \cos x = \cos(0) = 1$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x+1 = 0+1 = 1$

$\lim_{x \to 0} f(x) = 1$

3) $f(0) = 1 = \lim_{x \to 0} f(x)$

$f(x)$ is continuous at $x = 0$.

$x = 1$

1) $f(1) = 1+1 = 2$

2) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x+1) = 1+1 = 2$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (-x) = -1$

$\lim_{x \to 1} f(x)$ DNE

X

$x = 1$ is a discont. and it is a jump

② find the tangent line to $f(x) = 3e^x \tan x$ at $x = \frac{\pi}{4}$.

point-slope $\quad y - y_1 = m(x - x_1)$

$m =$ slope of f at $x = \pi/4$

(x_1, y_1) is a point on the curve $f(x)$.

$x_1 = \pi/4 \qquad y_1 = f(\pi/4) = 3e^{\pi/4}$

$$f'(x) = \frac{d}{dx}\left[3e^x \tan x\right] = 3e^x \tan x + 3e^x \sec^2 x$$

$$m = f'(\pi/4) = 3e^{\pi/4} + 3e^{\pi/4}\left(\frac{1}{\cos(\pi/4)}\right)^2$$

$$= 3e^{\pi/4} + 3e^{\pi/4}\left(\frac{1}{\sqrt{2}/2}\right)^2$$

$$= 3e^{\pi/4} + 3e^{\pi/4}(2)$$

$$m = 9e^{\pi/4}$$

$$y - 3e^{\pi/4} = 9e^{\pi/4}(x - \pi/4)$$

$$y = 9e^{\pi/4}x - \frac{9}{4}\pi e^{\pi/4} + 3e^{\pi/4}$$ ✓

③ Consider $f(x) = \frac{1}{2x-1}$, set-up the limit which computes the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \left[\frac{\frac{1}{2(x+h)-1} - \frac{1}{2x-1}}{h} \right]$$

$$= \lim_{h \to 0} \left[\frac{\frac{2x-1 - (2(x+h)-1)}{(2(x+h)-1)(2x-1)} \cdot \frac{1}{h}}{h \cdot \frac{1}{h}} \right]$$

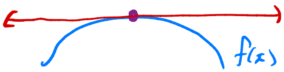
$$= \lim_{h \to 0} \frac{2x-1-(2(x+h)-1)}{h(2(x+h)-1)(2x-1)}$$

$$= \lim_{h \to 0} \frac{2x-1-2x-2h+1}{h(2x+2h-1)(2x-1)}$$

$$= \lim_{h \to 0} \frac{-2h}{h(2x+2h-1)(2x-1)}$$

$$= \lim_{h \to 0} \frac{-2}{(2x+2h-1)(2x-1)}$$

④ For what values of x does the function $f(x) = x^3 - 3x$ have horizontal tangent lines?



← horizontal tan line

Slope $= 0$

when is $f'(x) = 0$? so $f'(x) = 0$ solve for x