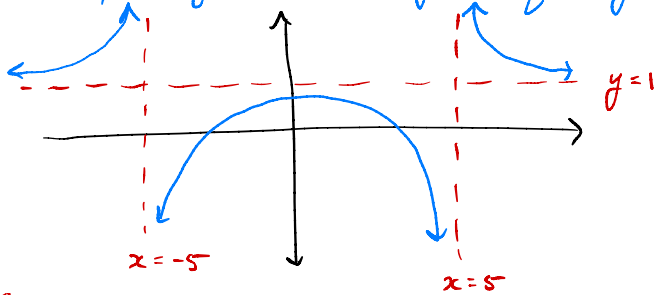


## Exam 2 Review

Exercise from last class.

find vertical / horiz / slant asym. of  $y = \frac{x^2+1}{x^2-25}$



vertical: let denominator = 0

$$x^2 - 25 = 0$$
$$x^2 = 25$$
$$x = \pm 5$$

two vertical asym. at  $x=5$  and  $x=-5$ .

horiz asym.

$$\lim_{x \rightarrow \infty} \frac{x^2+1}{x^2-25} = \lim_{x \rightarrow \infty} \frac{x^2}{x^2} = \lim_{x \rightarrow \infty} 1 = 1$$

horiz asym at  $y=1$ .

slant: deg of top > deg of bot

no slant asym.

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**Chain Rule:**  $y = f(g(x)) = (f \circ g)(x)$

$$y' = f'(g(x)) \cdot g'(x)$$

**Implicit differentiation**

$$y' = \frac{dy}{dx} = \frac{d}{dx} [y]$$

①  $\ln(6x) = 2xy^2$  . find  $\frac{dy}{dx}$  .

$$\frac{d}{dx} [\ln(6x)] = \frac{d}{dx} [2xy^2]$$

$$f(u) = \ln u \quad f'(u) = \frac{1}{u}$$

$$g(x) = 6x \quad g'(x) = 6$$

$$\frac{1}{6x} \cdot 6 = \frac{d}{dx} [2x \cdot y^2]$$

$$\frac{1}{x} = 2y^2 + 2x \cdot \frac{d}{dx} [y^2]$$

$$\frac{1}{x} = 2y^2 + 2x \cdot 2(y) \cdot \frac{dy}{dx}$$

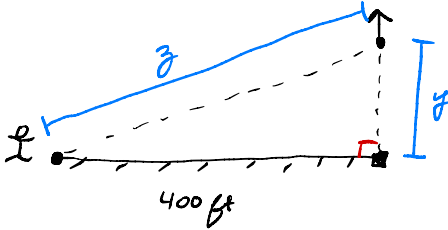
$$\frac{dy}{dx} = \frac{1 - 2xy^2}{4x^2y}$$

✓

$$f(u) = u^2$$

$$g(x) = y$$

② Person is standing 400 ft away from where a balloon is launched upward from the ground. The balloon travels at a rate of 30 ft/sec straight upwards. How fast is the distance between the person and the balloon increasing 10 seconds after launch?



$$400^2 + y^2 = z^2 \quad \frac{dy}{dt} = +30 \quad \frac{dz}{dt} \Big|_{t=10} = ?$$

$$\frac{d}{dt} [400^2 + y^2] = \frac{d}{dt} [z^2]$$

$$0 + 2y \cdot \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$\frac{y}{z} \frac{dy}{dt} = \frac{dz}{dt}$$

$$\frac{dz}{dt} \Big|_{t=10} = 30 \frac{y(t=10)}{z(t=10)}$$

$$\text{@ } t=10 \text{ sec} \quad y(t=10) = 30 \cdot 10 = 300 \text{ ft}$$

$$z(t=10) = \sqrt{400^2 + 300^2} = 500 \text{ ft}$$

$$\frac{dz}{dt} \Big|_{t=10} = 30 \cdot \frac{300}{500} = 30 \cdot \frac{3}{5} = 6 \cdot 3 = 18 \text{ ft/sec.}$$

③ find abs. ext. of  $y = \frac{x^2}{x+2}$

find the abs min on  $(-2, 2]$ .  
 $-2 < x \leq 2$

1) Find rel min.

2) decide which of the points above and  $x=2$  are the smallest.

$$1) y' = \frac{2x(x+2) - x^2(1)}{(x+2)^2}$$

Critical #s

$y' = 0$	$y'$ DNE
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$$0 = 2x(x+2) - x^2$$

$$= x^2 + 4x$$

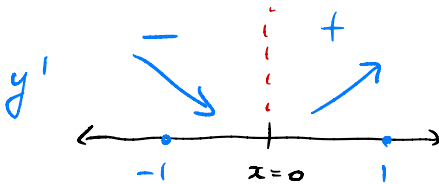
$$0 = x(x+4)$$

$$x = 0 \quad x = -4$$

~~$x = -4$~~   
not in  $(-2, 2]$

$y'$  exists everywhere  
on  $(-2, 2]$

Critical #  $x=0$ .



$x=0$  is rel min

$$y'(-1) < 0$$

$$y'(1) > 0$$

$x$	0	2
$y(x)$	0	1

$(0, 0)$  abs. min



## Exam #2 Review

①  $\ln(6x) = 2xy^2$  Find  $\frac{dy}{dx}$ .

$$\frac{d}{dx} [\ln(6x)] = \frac{d}{dx} [2xy^2]$$

$$f(u) = \ln(u) \quad f'(u) = \frac{1}{u}$$

$$g(x) = 6x \quad g'(x) = 6$$

$$\frac{1}{6x} \cdot 6 = \frac{d}{dx} [2x \cdot y^2]$$

$$\frac{1}{x} = 2y^2 + 2x \cdot \frac{d}{dx} [y^2]$$

$$\frac{1}{x} = 2y^2 + 2x \cdot 2(y) \cdot \frac{dy}{dx}$$

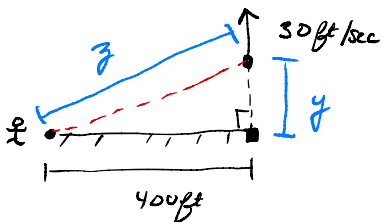
$$\frac{dy}{dx} = \frac{1 - 2xy^2}{4x^2y}$$

$$f(u) = u^2$$

$$g(x) = y$$



②



How fast is the distance between the person and the balloon increasing 10 seconds after launch?

$$400^2 + y^2 = z^2 \quad \frac{dy}{dt} = +30 \quad \frac{dz}{dt} \Big|_{t=10} = ??$$

$$\frac{d}{dt} [400^2 + y^2] = \frac{d}{dt} [z^2]$$

$$0 + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{y}{z} \frac{dy}{dt}$$

$$\frac{dz}{dt} \Big|_{t=10} = \frac{y|_{t=10}}{z|_{t=10}} \cdot 30$$

$$\text{@ } t=10; \quad y|_{t=10} = 30 \cdot 10 = 300 \text{ ft}$$

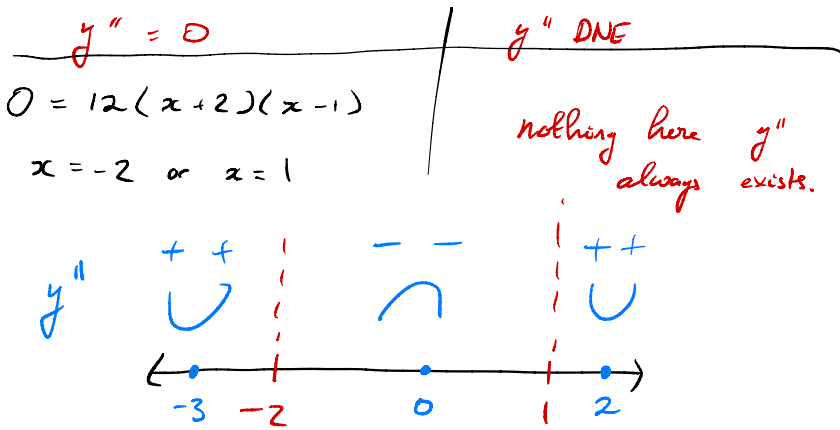
$$z|_{t=10} = \sqrt{400^2 + 300^2} = 500 \text{ ft}$$

$$\frac{dz}{dt} \Big|_{t=10} = \frac{300}{500} \cdot 30 = \frac{3}{5} \cdot 30 = 3 \cdot 6 = 18 \text{ ft/sec.}$$

③  $y = x^4 + 2x^3 - 12x^2 + 12x - 36$  find inflection pts.

$$y' = 4x^3 + 6x^2 - 24x + 12$$

$$\begin{aligned} y'' &= 12x^2 + 12x - 24 \\ &= 12(x^2 + x - 2) \\ &= 12(x+2)(x-1) \end{aligned}$$



$$y''(-3) = 12(-1)(-4) > 0$$

$$y''(0) = 12(2)(-1) < 0$$

$$y''(2) = 12(4)(1) > 0$$

Concave up  $(-\infty, -2)$  and  $(1, \infty)$   
 Concave down  $(-2, 1)$

Inflection pts:  $(-2, -108)$   $(1, -33)$  ✓



$$-2.5 \leq x \leq 2$$

④  $f(x) = x^3 + 3x^2 - 1$      $[-2.5, 2]$   
find abs. extrema.

$$\begin{aligned} f'(x) &= 3x^2 + 6x \\ &= 3x(x+2) \end{aligned}$$

$f' = 0$	$f'$ DNE
$0 = 3x(x+2)$	<i>nothing here.</i>
$x=0 \quad x=-2$	

$x$	$f(x)$	
-2.5	2.125	
-2	3	
0	-1	← (0, -1) abs min
2	19	← (2, 19) abs max