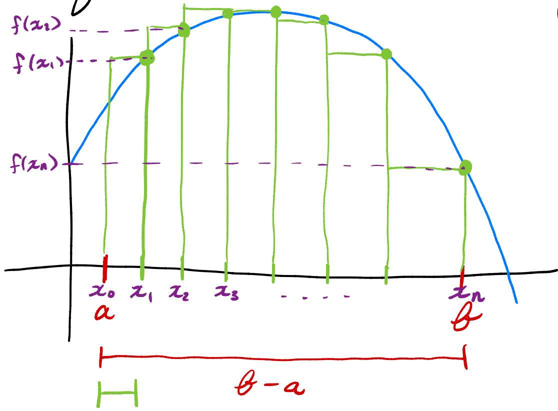
Exam # 3 Review

Riemann Sums of $f(x)$ on $[a,b]$ using n rectangles.

Right :

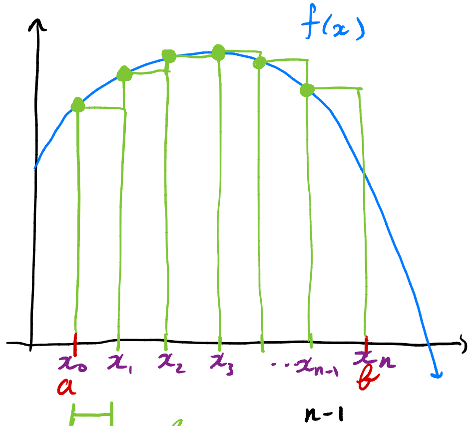


$x_i = a + i\Delta x$

$\Delta x := \frac{b-a}{n}$

$$\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} f(a+i\Delta x)\Delta x$$

Left

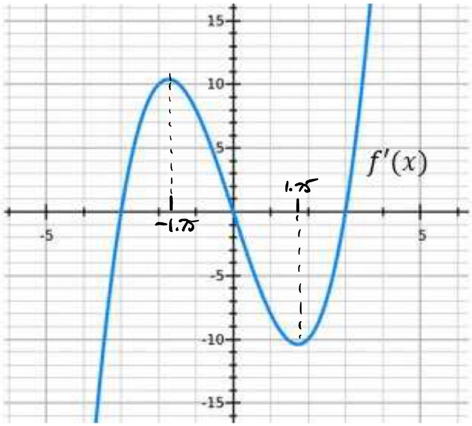


$x_i = a + i\Delta x$

$\Delta x = \frac{b-a}{n}$

$$\sum_{i=0}^{n-1} f(x_i)\Delta x = \sum_{i=0}^{n-1} f(a+i\Delta x)\Delta x$$

1



Crit #'s: $x = -3, 0, 3$

Inc: $(-3, 0)$, $(3, \infty)$

Dec: $(-\infty, -3)$, $(0, 3)$

Rel min: $x = -3, 3$

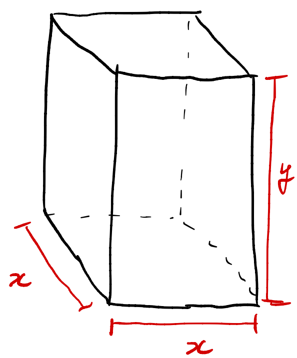
Rel max: $x = 0$

Concave up: $(-\infty, -1.75)$
$(1.75, \infty)$

Concave down: $(-1.75, 1.75)$

Inflection pts: $x = -1.75, 1.75$.

(2) A rectangular box has a square base
Sum of height and perimeter of base is 12 in
What is the max volume?



Obj: $V = x^2 y$

Const: $12 = y + 4x$; $x > 0$; $y > 0$

$y = 12 - 4x$

$y > 0$
$12 - 4x > 0$
$-4x > -12$
$x < 3$

Obj: $V = x^2(12 - 4x) = 12x^2 - 4x^3$

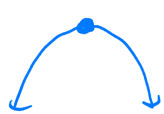
Const: $0 < x < 3$

Find abs max of $V = 12x^2 - 4x^3$ on $(0, 3)$

Crit #'s

$V' = 24x - 12x^2$
$0 = 24x - 12x^2$
$0 = -12x(x - 2)$
$\cancel{x = 0}$ $\quad x = 2$
not in $(0,3)$

$V'' = 24 - 24x$
$V''(2) = 24 - 24(2) < 0$



rel max!

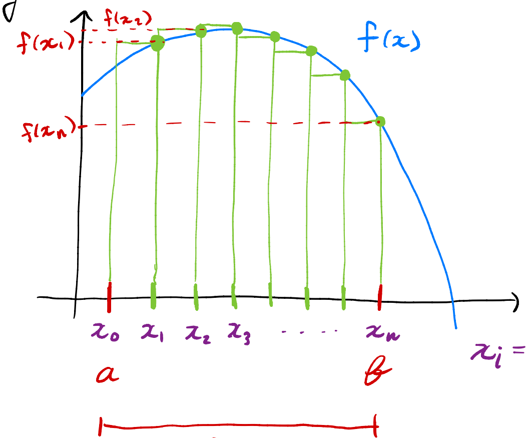
Only rel max, so abs max

max volume $V(2) = (2)^2(12 - 4(2)) = \underline{16 \text{ in}^3}$.

Exam #3 Review

Riemann Sums: What is approx signed area of $f(x)$ on $[a,b]$ using n rectangles

Right:

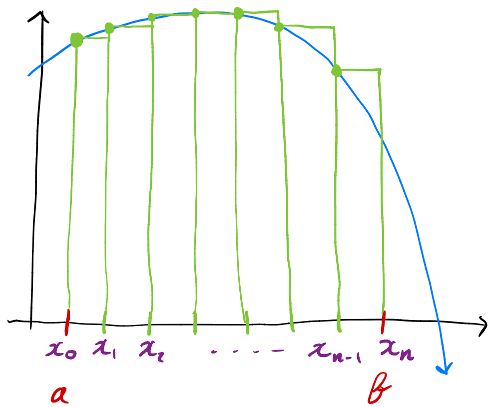


$$\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} f(a+i\Delta x)\Delta x$$

$x_i = a + i\Delta x$

$\Delta x := \dfrac{b-a}{n}$

Left:



$$\sum_{i=0}^{n-1} f(x_i)\Delta x = \sum_{i=0}^{n-1} f(a+i\Delta x)\Delta x$$

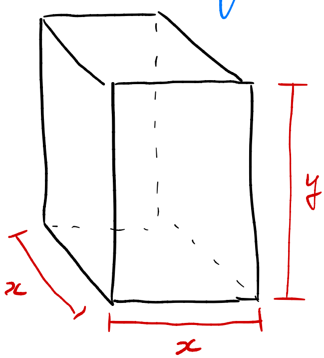
$x_i = a + i\Delta x$

$\Delta x = \dfrac{b-a}{n}$

① A box with a square base will be built for \$48.

Material for top and bot \$2 / ft^2
Material for sides \$1 / ft^2

What is largest volume of box?



Obj: $V = x^2 y$

Const: $48 = 2(2x^2) + 1(4xy)$; $x > 0$; $y > 0$.

$$48 = 4x^2 + 4xy$$
$$12 = x^2 + xy$$
$$xy = 12 - x^2$$
$$y = \frac{12 - x^2}{x}$$

$y > 0$ $x > 0$

$$\frac{12 - x^2}{x} > 0$$
$$12 - x^2 > 0$$
$$-x^2 > -12$$
$$x^2 < 12$$
$$-\sqrt{12} < x < \sqrt{12}$$

Obj: $V = x^2 \left(\frac{12 - x^2}{x} \right) = x(12 - x^2) = 12x - x^3$

Const: $0 < x < \sqrt{12}$

Find the abs max of $V = 12x - x^3$ on $(0, \sqrt{12})$.

2. $f(x) = \dfrac{x^2+3x+2}{x^2-1}$ find vert / horz / slant.

vert:

$$f(x) = \frac{(x+2)\cancel{(x+1)}}{(x-1)\cancel{(x+1)}} = \frac{x+2}{x-1}$$ w/ hole at $x=-1$

$x-1=0$

$x=1$ ← vert asym.

horz: $\displaystyle\lim_{x\to\infty} \frac{x^2+3x+2}{x^2-1} = \lim_{x\to\infty} \frac{x^2}{x^2} = \lim_{x\to\infty} 1 = 1$

horz asy at $y=1$.

slant: no slant.

for a slant: need

deg of top = deg bot + 1