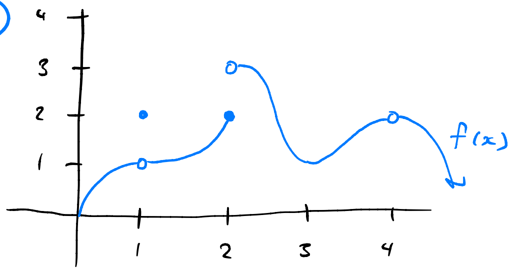
Final Review I

(3)



I. $f(x)$ disc. at $x=1$, $x=2$, $x=4$ True

II. $\lim_{x \to 2} f(x) = 2$ False

III. $\lim_{x \to 4} f(x)$ DNE False

IV. $\lim_{x \to 1} f(x) = 2$ False

(9) $f(x) = \frac{1}{x+1}$ $g(x) = \frac{x-1}{x^2-1}$

A. $f(x)$ has a vert. asym. at $x = -1$ $\frac{1}{0}$ True

B. $\lim_{x \to -1} g(x) = ?$ $\frac{-1-1}{(-1)^2-1} = \frac{-2}{0}$

x	-1.1	-1.01	-1	$-.99$	$-.9$
$g(x)$	-10	-100	$\times$	100	10

asym.

$\lim_{x \to -1} g(x)$ DNE

35 $f(x) = \begin{cases} 2\cos x & x \le 0 \\ x+2 & 0 < x < 2 \\ 3 & x \geq 2 \end{cases}$

Find and classify all discont of f.

continuity : (i) $f(c)$ exist

(ii) $\lim_{x\to c} f(x)$ exist (left limit = right limit)

(iii) $\lim_{x\to c} f(x) = f(c)$

$x=0$

i) $f(0) = 2\cos(0) = 2$ ✓

ii) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 2\cos x = 2\cos(0) = 2$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x+2 = 0+2 = 2$

$\lim_{x\to 0} f(x) = 2$ ✓

iii) ✓

$x=2$

i) $f(2) = 3$ ✓

ii) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x+2 = 4$

$\neq$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 3 = 3$

$\lim_{x\to 2} f(x)$ DNE ✗

jump discont.

(17) $f(x) = \frac{1}{2x-1}$. Find $f'(x)$ using definition.

Defn. The derivative of $f(x)$

$$f'(x) := \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2(x+h)-1} - \frac{1}{2x-1}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2x-1-(2(x+h)-1)}{(2(x+h)-1)(2x-1)}}{h} \qquad \frac{\frac{1}{h}}{1/h}$$

$$= \lim_{h\to 0} \frac{2x-1-(2(x+h)-1)}{h(2(x+h)-1)(2x-1)}$$

$$= \lim_{h\to 0} \frac{\cancel{2x}\cancel{-1}-\cancel{2x}-2\cancel{h}\cancel{+1}}{\cancel{h}(2x+2h-1)(2x-1)}$$

$$= \lim_{h\to 0} \frac{-2}{(2x+2h-1)(2x-1)}$$

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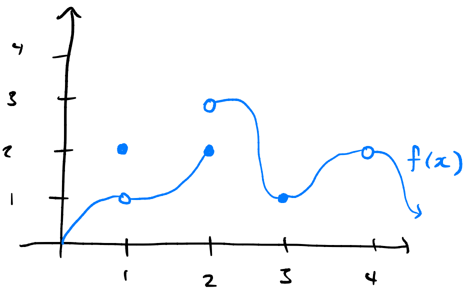

I. $f(x)$ is discont at $x=1$, $x=2$, $x=4$ True

II. $\lim_{x\to 2} f(x) = 2$ False (1-sided limits not equal)

III. $\lim_{x\to 4} f(x)$ DNE False $\lim_{x\to 4} f(x) = 2$

IV. $\lim_{x\to 1} f(x) = 2$ False $\lim_{x\to 1} f(x) = 1$

(35)

$$f(x) = \begin{cases} 2\cos x & x \leq 0 \\ x+2 & 0 < x < 2 \\ 3 & x \geq 2 \end{cases}$$

Find / classify all discont of f.

Defn. $f(x)$ is cont. at $x=c$ if

i) $f(c)$ must exist

ii) $\lim_{x\to c} f(x)$ exist $\left(\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) \right)$

iii) $\lim_{x\to c} f(x) = f(c)$

$\underline{x=0}$ i) $f(0) = 2\cos(0) = 2$ ✓

ii) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 2\cos x = 2\cos(0) = 2$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x+2 = 0+2 = 2$

$$\lim_{x\to 0} f(x) = 2 \quad ✓$$

iii) ✓

f is cont. at $x = 0$.

$x = 2$ | i) $f(2) = 3$ ✓

ii) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x+2 = 2+2 = 4$ *

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 3 = 3$

$\lim_{x\to 2} f(x)$ DNE ✗

$x = 2$ is a jump discont.

(39) Particle is moving in a straight given by
$$s(t) = 3t^2 - 12t + 9$$
in meters and t in minutes.
At what time is velocity $= 0$.

$v(t) = s'(t) = 6t - 12$

$0 = 6t - 12$
$6t = 12$
$t = 2$ minutes.

s position
$\frac{d}{dt} \downarrow \uparrow \int \cdot dt$
v velocity
$d/dt \downarrow \uparrow \int \cdot dt$
a acceleration

53 pos. $s(t) = \frac{7}{3}t^3 - 7t^2 - t + 16$ (ft $t =$ sec.)

What is pos. when velocity is 20ft/sec.

$v(t) = s'(t) = 7t^2 - 14t - 1$

$$20 = 7t^2 - 14t - 1$$
$$0 = 7t^2 - 14t - 21$$
$$0 = t^2 - 2t - 3$$
$$0 = (t-3)(t+1)$$
$$t = 3, \cancel{-1}$$

$$s(3) = \frac{7}{3}(3)^3 - 7(3)^2 - 3 + 16$$
$$= 7\cdot 9 - 7\cdot 9 - 3 + 16$$
$$= 13 \text{ ft} \checkmark$$

38 $f(x) = 6\sin x$. Find tangent line at $x = \frac{\pi}{3}$

$$y - y_1 = m(x - x_1)$$

$m =$ "slope at $x = \pi/3$" $= \left.\frac{df}{dx}\right|_{x=\frac{\pi}{3}} = 6\cos\left(\frac{\pi}{3}\right) = 3$

(x_1, y_1) is a point on the tangent line.

$\left(\frac{\pi}{3}, f\left(\frac{\pi}{3}\right)\right) = \left(\frac{\pi}{3}, 3\sqrt{3}\,?\right)$