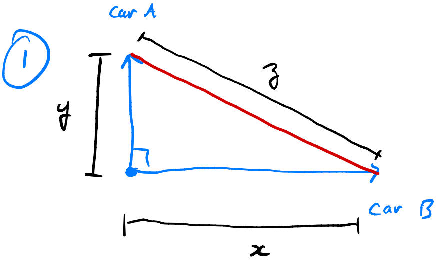
Final Review II

①



After 2 hours is 60 mi from starting pt and travelling at 40mph.
Car B is 80 miles to the east and travelling at 55 mph.

At what rate is the distance between the cars changing at this moment?

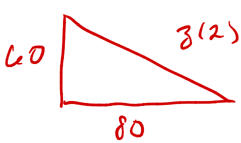
$$x^2 + y^2 = z^2 \qquad \text{goal}: \left.\frac{dz}{dt}\right|_{t=2}$$

$$\left.\frac{dx}{dt}\right|_{t=2} = 55 \qquad \left.\frac{dy}{dt}\right|_{t=2} = 40 \qquad x(2) = 80 \qquad y(2) = 60$$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$2\,x(2) \cdot \left.\frac{dx}{dt}\right|_{t=2} + 2\,y(2) \cdot \left.\frac{dy}{dt}\right|_{t=2} = 2\,z(2)\left.\frac{dz}{dt}\right|_{t=2}$$

$$2 \cdot 80 \cdot 55 + 2 \cdot 60 \cdot 40 = 2\,z(2)\left.\frac{dz}{dt}\right|_{t=2}$$

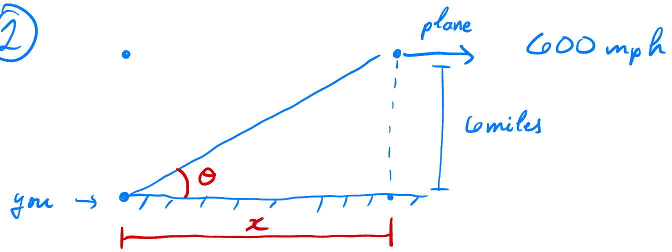


$$z(2) = \sqrt{60^2 + 80^2} = \sqrt{10000} = 100$$

$$8800 + 4800 = 2 \cdot 100 \cdot \left.\frac{dz}{dt}\right|_{t=2}$$

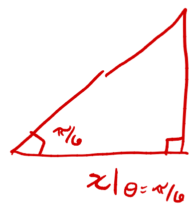
$$\left.\frac{dz}{dt}\right|_{t=2} = 68 \text{ mph}$$

(2)



Find rate of change of angle of elevation when the angle is $\pi/6$.

$$\frac{dx}{dt} = 600 \qquad \left.\frac{d\theta}{dt}\right|_{\theta=\frac{\pi}{6}} = ??$$



$$\tan\theta = \frac{6}{x} = 6x^{-1}$$

$$\xrightarrow{d/dt} -6x^{-2}\cdot\frac{dx}{dt}$$

$$\sec^2\theta\cdot\frac{d\theta}{dt} = \frac{-6}{x^2}\cdot\frac{dx}{dt}$$

$$\tan\frac{\pi}{6} = \frac{6}{x}$$

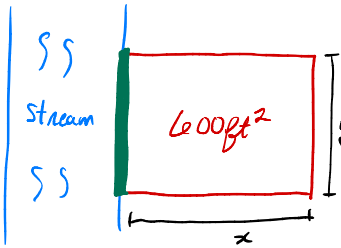
$$x = 6/\tan\pi/6 = 6\sqrt{3}$$

$$\sec^2\left(\frac{\pi}{6}\right)\cdot\left.\frac{d\theta}{dt}\right|_{\theta=\pi/6} = \frac{-6}{(x|_{\theta=\pi/6})^2}\cdot 600$$

$$\frac{4}{3}\cdot\left.\frac{d\theta}{dt}\right|_{\theta=\pi/6} = \frac{-6}{(6\sqrt{3})^2}\cdot 600$$

$$\left.\frac{d\theta}{dt}\right|_{\theta=\pi/6} = -\frac{3\cdot 6\cdot 600}{4\cdot 36\cdot 3} = -\frac{10800}{432} = -25 \text{ rad/h}$$

③



\$20 / ft for waterproof fence

\$10 / ft for wood fence

What is the min total cost for the fencing?

Obj : $C = 10(2x + y) + 20y$

Const : $600 = xy; \quad x > 0; \quad y > 0$

$y = \frac{600}{x}$

Obj : $C = 10\left(2x + \frac{600}{x}\right) + 20 \cdot \frac{600}{x}$

$= 20x + \frac{18000}{x}$

Const : $0 < x < +\infty$

$y > 0$

$\frac{600}{x} > 0$

$\frac{1}{x} > 0$

goal: find abs. min. of $C = 20x + 18000/x$ on $(0, \infty)$

$C' = 20 - 18000/x^2$

Crit #'s

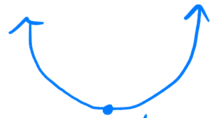
$C' = 0$	C' DNE
$20 = \frac{18000}{x^2}$	$\cancel{x = 0}$
$x^2 = \frac{1800}{2}$	not in $(0, \infty)$

$$x = \pm 30$$

-30 not in $(0, \infty)$

Crit # is $x = 30$.

$$C'' = \frac{36000}{x^3} \qquad C''(30) > 0$$



← rel min ✓

Abs rel min on $(0, \infty)$

x	C
30	$C(30) = 20(30) + \frac{18000}{30} = 600 + 600 = 1200$

min cost to build fence is \$1200.

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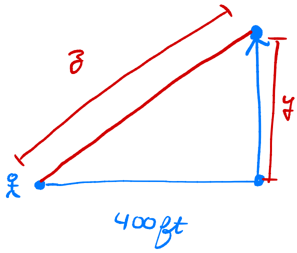


How fast is the Balloon moving away from us 10 sec after launch?

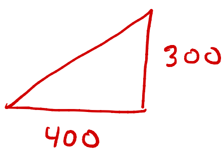
$$\frac{dy}{dt} = 30 \qquad \left.\frac{dz}{dt}\right|_{t=10} = ??$$

$$400^2 + y^2 = z^2$$

$$0 + 2y \cdot \frac{dy}{dt} = 2z\,\frac{dz}{dt}$$

$$2y(10)\cdot 30 = 2z(10)\cdot \left.\frac{dz}{dt}\right|_{t=10}$$

$$y(10) = 30 \cdot 10 = 300$$

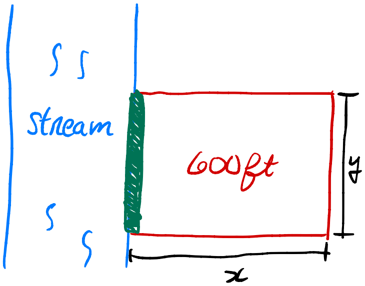


$$z(10) = \sqrt{400^2 + 300^2} = 500$$

$$2 \cdot 300 \cdot 30 = 2 \cdot 500 \cdot \left.\frac{dz}{dt}\right|_{t=10}$$

$$\left.\frac{dz}{dt}\right|_{t=10} = 18 \text{ ft/sec}$$

(2)



\$20/ft for waterproof fence

\$10/ft for wood fence

Find minimal total cost of fencing.

Obj : $C = 10(2x + y) + 20y$

Const : $600 = xy$; $x > 0$; $y > 0$

$$y = \frac{600}{x}$$

Obj : $C = 10\left(2x + \frac{600}{x}\right) + 20 \cdot \frac{600}{x}$

$$= 20x + \frac{18000}{x}$$

Const : $0 < x < +\infty$

$y > 0$

$\frac{600}{x} > 0$

$\frac{1}{x} > 0$

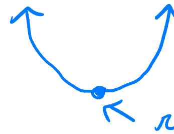
goal: find abs min of $C = 20x + \frac{18000}{x}$ on $(0, \infty)$.

$$C' = 20 - \frac{18000}{x^2}$$

Crit #'s

$C' = 0$	C' DNE
$20 = 18000/x^2$	$\cancel{x = 0}$
$x^2 = 900$	
$x = 30, \cancel{-30}$	

$$C'' = \frac{36\,000}{x^3} \qquad C''(30) > 0$$



rel min ✓

all rel min of C on $(0, \infty)$.

x	C
30	$C(30) = 20(30) + 18000/30 = 1200$

min cost to build the fence is \$1200.

(3) $$\int_{-\pi/3}^{\pi/3} 9 \tan x \cos x + 9 \, dx$$

$$\int f(x)\, g(x)\, dx \neq \int f(x)\, dx \int g(x)\, dx$$

$$\tan x \cos x = \frac{\sin x}{\cos x} \cdot \cos x = \sin x$$

$$\int_{-\pi/3}^{\pi/3} 9 \sin x + 9 \, dx \overset{FTC}{=} (-9\cos x + 9x)\Big|_{-\pi/3}^{\pi/3}$$

$$= -9\cos\left(\frac{\pi}{3}\right) + 9\left(\frac{\pi}{3}\right) - \left[-9\cos\left(-\frac{\pi}{3}\right) + 9\left(-\frac{\pi}{3}\right)\right]$$

$$= -9/2 + 3\pi + 9/2 + 3\pi$$

$$= 6\pi$$

(4) $\int_{-9}^{4} f(x)\,dx = -4$

$\int_{-6}^{-11} f(x)\,dx = 0$

$\int_{-11}^{4} f(x)\,dx = -13$

$$\int_a^c = \int_a^b + \int_b^c$$

$$\int_a^b = -\int_b^a$$

a) $\int_{-6}^{4} f(x)\,dx$

$= \int_{-6}^{-11} f(x)\,dx + \int_{-11}^{4} f(x)\,dx$

$= 0 + -13$

$= -13$

b) $\int_{4}^{-9} f(x)\,dx = -\int_{-9}^{4} f(x)\,dx = -(-4) = 4$