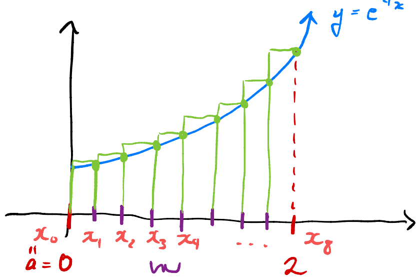
Final Review III

Exam 3 practice #14 Use right Riemann sums to approximate the area under $y = e^{4x}$ on $[0,2]$ with 8 rectangles.



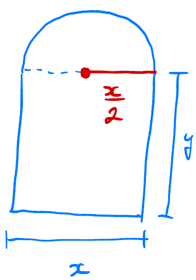
$\Delta x = \dfrac{2-0}{8} = \dfrac{1}{4}$

$$\sum_{i=1}^{8} e^{4x_i}\,\Delta x = \sum_{i=1}^{8} e^{4(i/4)}\left(\frac{1}{4}\right)$$

$$x_i = 0 + i\Delta x = i/4$$

$$= \sum_{i=1}^{8} \frac{1}{4} e^{i} \quad \checkmark$$

Exam 3 practice 37



Find x so that the area of the window is maximized if the total perimeter is 10 ft.

$$\text{Obj}: \quad A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 = xy + \frac{\pi}{8}x^2$$

$$\text{Const}: \quad 10 = x + y + \frac{\pi}{2}x + y$$

$$10 = (1 + \pi/2)x + 2y \;; \qquad x > 0 \;; \quad \underline{y > 0}$$

$$y = \frac{1}{2}\left(10 - (1+\pi/2)x\right) = 5 - (1/2 + \pi/4)x$$

$$5 - \left(\frac{1}{2} + \frac{\pi}{4}\right)x > 0$$

$$\left(\frac{1}{2} + \frac{\pi}{4}\right)x < 5$$

$$x < \frac{20}{2+\pi}$$

$$\text{Obj}: \quad A = x\left(5 - (1/2 + \pi/4)x\right) + \frac{\pi}{8}x^2 = \left(\frac{\pi}{8} - \frac{1}{2} - \frac{\pi}{4}\right)x^2 + 5x$$

Const : $0 < x < \dfrac{20}{2+\pi}$

After exam 3 practice # 4

We deposite \$1000 into a savings account which compounds interest continuously at an annual rate of 5%, how many years will it take for the money to double?

P = ammount in the account

t = time in years

$k = 0.05$

$P(0) = ce^{k \cdot 0} = c$

$$P(t) = ce^{kt} = 1000e^{0.05t}$$

$$2000 = P(t) = 1000\,e^{0.05t}$$

$$2 = e^{0.05t}$$

$$\ln(2) = 0.05\,t$$

$$0.05 = \frac{5}{100} = \frac{1}{20}$$

$$\ln(2) = \frac{1}{20}\,t$$

$$t = 20\ln 2$$

✓

After exam 3 practice #8 I

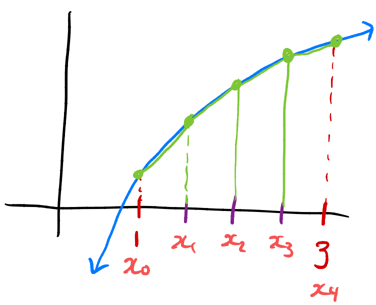
Use trapezoid rule to approximate

$$\int_1^3 \ln(x^2+3)\,dx$$

with 4 trapezoids

$$\Delta x = \frac{3-1}{4} = \frac{1}{2}$$

$$x_i = 1 + i/2$$



$$\Rightarrow T_4 = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$$

$$T_4 = \frac{1}{4}\Big(\ln\left((1)^2+3\right) + 2\ln\left(\left(\tfrac{3}{2}\right)^2+3\right) + 2\ln\left((2)^2+3\right) + 2\ln\left(\left(\tfrac{5}{2}\right)^2+3\right) + \ln\left((3)^2+3\right) \Big]$$

$$= \frac{1}{4}\left(\ln(4) + 2\ln\left(\frac{21}{4}\right) + 2\ln(7) + 2\ln\left(\frac{37}{4}\right) + \ln(12)\right) \checkmark$$

Final Review III

After exam 3 practice # 51

^{240}Pu has a half life of 6563 years
what is the decay rate k?

P = ammount of ^{240}Pu after t years.

$$\frac{dP}{dt} = kP \rightsquigarrow P(t) = ce^{-kt}$$

$$\frac{c}{2} = P(6563) = ce^{-k \cdot 6563}$$

$$1/2 = e^{-6563k}$$

$$\ln(1/2) = -6563k$$

$$k = \frac{\ln 2}{6563} \checkmark$$

Exam 3 practice #10

If a product is sold for p dollars per unit they can sell $q = 2800 - 200p$ units.

Each unit costs \$10 to make.
What is the maximum profit the company can make?

Obj : $P = (\text{Revenue}) - (\text{expences})$

$$= p(2800 - 200p) - 10(2800 - 200p)$$

$$= -200p^2 + 4800p - 28000$$

Const : $0 < p < +\infty$

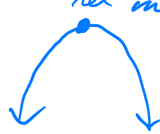
$$P' = -400p + 4800$$

Crit #'s

$P' = 0$	or P' DNE
$400p = 4800$ $4p = 48$ $p = 12$	✗

$P'' = -400$ $\quad$ $P''(12) = -400 < 0$

rel max ✓



All rel max of P on $(0, \infty)$

p	$P(p)$
12	$P(12) = -200(12)^2 + 4800(12) - 28000$ $= 800$

We maximize profit when we sell for \$12

Exam 1 practice # 35

$$f(x) = \begin{cases} 2\cos x & x \le 0 \\ x+2 & 0 < x < 2 \\ 3 & x \geq 2 \end{cases}$$

Find the discontinuities of $f(x)$.

Recall: f is cont. at $x = c$ if

i) $f(c)$ exists

ii) $\lim_{x \to c} f(x)$ exists

iii) $f(c) = \lim_{x \to c} f(x)$

$x = 0$ | i) $f(0) = 2$ ✓

ii) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 2\cos x = 2\cos(0) = 2$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x+2 = 0+2 = 2$$

$$\lim_{x\to 0} f(x) = 2 \quad \checkmark$$

iii) $f(0) = 2 = \lim_{x\to 0} f(x) \quad \checkmark$

f is cont at $x=0$.

$x=2$ | i) $f(2) = 3 \quad \checkmark$

ii) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x+2 = 2+2 = 4$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 3 = 3$$

$\lim_{x\to 2} f(x)$ DNE

f is a jump discont. at $x=2$