# Lecture 12: Higher order derivatives

The second derivative

$$\frac{d^2}{dx^2}\left[f(x)\right] = \frac{d}{dx}\left[\frac{d}{dx}\left[f(x)\right]\right]$$

other notation: $f''(x)$ or $f^{(2)}(x)$

we also have the third derivative:

$$\frac{d^3}{dx^3}\left[f(x)\right] = \frac{d}{dx}\left[\frac{d}{dx}\left[\frac{d}{dx}\left[f(x)\right]\right]\right]$$

other notation: $f'''(x)$ or $f^{(3)}(x)$

the nth derivative

$$\frac{d^n}{dx^n}\left[f(x)\right] = \underbrace{\frac{d}{dx}\left[\cdots \frac{d}{dx}\right.}_{n \text{ derivatives}}\left.\left[f(x)\right]\cdots\right]$$

other notation: $f^{(n)}(x)$.

e.g. ① $y = x^3 + 2x$; find $y''$.

$$y'' = \frac{d}{dx}\left[\frac{d}{dx}[y]\right] = \frac{d}{dx}[y']$$

$$y' = \frac{d}{dx}[y] = \frac{d}{dx}[x^3 + 2x] = 3x^2 + 2$$

$$y'' = \frac{d}{dx}[3x^2 + 2] = 6x + 0 = 6x$$

② $g(x) = \underbrace{6e^{5x}}\,\underbrace{\cos(2x)}$ find $g''(x)$.

first need $g'(x)$.

$$g'(x) = \frac{d}{dx}[6e^{5x}]\cos(2x) + 6e^{5x}\frac{d}{dx}[\cos(2x)]$$

$$= 6 \cdot e^{5x} \cdot 5 \cdot \cos(2x) + 6e^{5x}(-\sin(2x)) \cdot 2$$

$$= 30e^{5x}\cos(2x) - 12e^{5x}\sin(2x)$$

$$g''(x) = \frac{d}{dx}[30e^{5x}\cos(2x) - 12e^{5x}\sin(2x)]$$

$$= \frac{d}{dx}[30e^{5x}] \cdot \cos(2x) + 30e^{5x}\frac{d}{dx}[\cos(2x)]$$

$$+ \frac{d}{dx}[-12e^{5x}]\sin(2x) + -12e^{5x}\frac{d}{dx}[\sin(2x)]$$

$$= 30 \cdot e^{5x} \cdot 5 \cdot \cos(2x) + 30e^{5x}(-\sin(2x)) \cdot 2$$
$$- 12e^{5x} \cdot 5 \cdot \sin(2x) - 12e^{5x}\cos(2x) \cdot 2$$

$$= 126 e^{5x} \cos(2x) - 120 e^{5x} \sin(2x)$$

③ Suppose $f^{(4)}(x) = 10 \sec(2x)$. Find $f^{(5)}(x)$.

$$f^{(5)}(x) = \frac{d}{dx}\left[ f^{(4)}(x) \right]$$
$$= \frac{d}{dx}\left[ 10 \sec(2x) \right]$$
$$= 10 \sec(2x) \tan(2x) \cdot 2$$
$$= 20 \sec(2x) \tan(2x)$$

④ Let $s(t) = t^3 + 10t^2 - t + 1$ be the position of a particle moving in a straight line. What is the acceleration of the particle?

acc.: $a(t) = s''(t)$ or $a(t) = v'(t)$

$$s'(t) = 3t^2 + 20t - 1 \quad \leftarrow \text{velocity}$$

$$a(t) = s''(t) = \frac{d}{dt}\left[ 3t^2 + 20t - 1 \right]$$
$$= 6t + 20 \quad \leftarrow \text{acceleration.}$$

# Lecture 12 : Higher order derivatives

## The second derivative

$$\frac{d^2}{dx^2}[f(x)] = \frac{d}{dx}\left[\frac{d}{dx}[f(x)]\right] = \frac{d}{dx}[f'(x)]$$

other notation : $f''(x)$ or $f^{(2)}(x)$

## third derivative

$$\frac{d^3}{dx^3}[f(x)] = \frac{d}{dx}\left[\frac{d}{dx}\left[\frac{d}{dx}[f(x)]\right]\right]$$

other notation : $f'''(x)$ or $f^{(3)}(x)$

## nth derivative

$$\frac{d^n}{dx^n}[f(x)] = \underbrace{\frac{d}{dx}\left[\cdots \frac{d}{dx}\right.}_{n \text{ derivatives}}\left.[f(x)]\cdots\right]$$

other notation : $f^{(n)}(x)$

e.g. ① Let $s(t) = t^3 + 10t^2 - t + 1$ be the position of a particle moving in a straight line in meters at time $t$ in seconds.

Find the acceleration of the particle.

acceleration $a(t) = s''(t) = v'(t)$

$$s'(t) = \frac{d}{dt}\left[t^3 + 10t^2 - t + 1\right]$$
$$= 3t^2 + 20t - 1 \quad \leftarrow \text{velocity}$$

$$a(t) = s''(t) = \frac{d}{dt}\left[3t^2 + 20t - 1\right]$$
$$= 6t + 20 \quad m/s^2$$

② $g(x) = 6e^{5x}\cos(2x)$   find $g''(x)$.

first need $g'(x)$.

$$g'(x) = \frac{d}{dx}\left[6e^{5x}\cos(2x)\right]$$
$$= \frac{d}{dx}\left[6e^{5x}\right]\cos(2x) + 6e^{5x}\frac{d}{dx}\left[\cos(2x)\right]$$
$$= 6e^{5x}\cdot 5\cos(2x) + 6e^{5x}\left(-\sin(2x)\cdot 2\right)$$
$$= 30e^{5x}\cos(2x) - 12e^{5x}\sin(2x)$$

$$g''(x) = \frac{d}{dx}\left[30e^{5x}\cos(2x) - 12e^{5x}\sin(2x)\right]$$
$$= \frac{d}{dx}\left[30e^{5x}\right]\cos(2x) + 30e^{5x}\frac{d}{dx}\left[\cos(2x)\right]$$
$$+ \frac{d}{dx}\left[-12e^{5x}\right]\sin(2x) + -12e^{5x}\frac{d}{dx}\left[\sin(2x)\right]$$
$$= 30e^{5x}\cdot 5\cos(2x) + 30e^{5x}\left(-\sin(2x)\cdot 2\right)$$
$$- 12e^{5x}\cdot 5\sin(2x) - 12e^{5x}\cos(2x)\cdot 2$$

$$= 126e^{5x}\cos(2x) - 120e^{5x}\sin(2x)$$

③ Suppose $f^{(4)}(x) = 10\sec(2x)$, find $f^{(5)}(x)$.

$$f^{(5)}(x) = \frac{d}{dx}\left[f^{(4)}(x)\right] = \frac{d}{dx}\left[10\sec(2x)\right]$$
$$= 10\sec(2x)\tan(2x)\cdot 2$$
$$= 20\sec(2x)\tan(2x).$$