Lecture 14: Related rates

e.g. ① Suppose x and y are functions of t
$7x^9y = 14$; $\frac{dx}{dt} = 2$. find $\frac{dy}{dt}$ at $x=1$.

$x = g(t)$ $\quad y = h(t)$

$$\frac{d}{dt}\left[7x^9 \cdot y\right] = \frac{d}{dt}[14]$$

$$\frac{d}{dt}\left[7x^9\right]\cdot y + 7x^9 \frac{d}{dt}[y] = 0$$

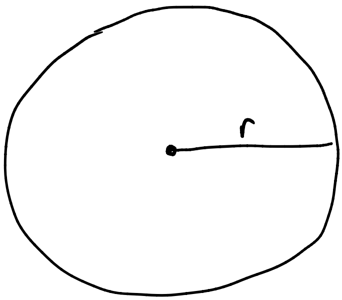
$$63x^8 \cdot \frac{dx}{dt} \cdot y + 7x^9 \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-63x^8 \cdot \frac{dx}{dt} \cdot y}{7x^9}$$

$7x^9y = 14$ @ $x = 1$ $\Rightarrow$ $7(1)^9 y = 14$
$y = 2$

$$\left.\frac{dy}{dt}\right|_{x=1} = \frac{-63(1)^8 \cdot 2 \cdot 2}{7(1)^9} = -36 \quad \checkmark$$

(2) A circle with radius, r, which is increasing at a rate of 9 cm/min. Find the R.o.C. of the circumference of the circle wrt time t in min when $r = 5$ cm.



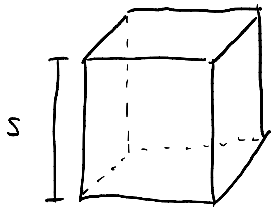
$$\frac{dr}{dt} = +9 \qquad \left.\frac{dC}{dt}\right|_{r=5} = ?$$

$C = 2\pi r$ ← circumference

$$\frac{dC}{dt} = \frac{d}{dt}[2\pi r]$$
$$= 2\pi \frac{d}{dt}[r]$$
$$= 2\pi \frac{dr}{dt}$$

$$\left.\frac{dC}{dt}\right|_{r=5} = 2\pi(9) = 18\pi \text{ cm/min}$$

(3) Suppose all the edges of a cube are shrinking at a rate of 35 cm / sec. How fast is surface area of the cube decreasing wrt. to time t in sec when the edge length is 11 cm?



A : surface area $\quad A = 6s^2$

s : length of an edge

$$\frac{ds}{dt} = -35 \qquad \left.\frac{dA}{dt}\right|_{s=11} = ?$$

$$\frac{dA}{dt} = \frac{d}{dt}\left[6s^2\right] = 12s \cdot \frac{ds}{dt}$$

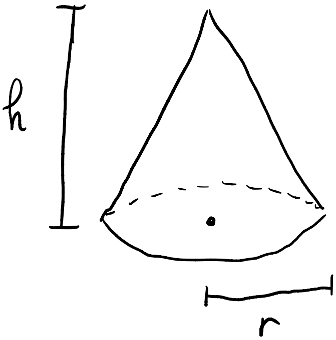
$$\left.\frac{dA}{dt}\right|_{s=11} = 12(11)\cdot(-35) = -4620 \text{ cm}^2/\text{sec}$$

Surface area is decreasing at 4620 cm²/sec when $s = 11$.

4. • Sand is being poured into a conical pile at a rate of 11 cm^3/sec

• length of the diameter of base = altitude

How fast is alt. changing wrt. time when pile is 3 cm high.



$V = \frac{1}{3}\pi r^2 h$ volume

$2r = h$

$r = h/2$

$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$

$\left.\frac{dh}{dt}\right|_{h=3} = ??$

$\frac{dV}{dt} = +11$

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$x = g(t) \quad y = h(t)$

$$\frac{d}{dt}\left[7x^9 \cdot y\right] = \frac{d}{dt}[14]$$

$$\frac{d}{dt}\left[7x^9\right]\cdot y + 7x^9 \frac{d}{dt}[y] = 0$$

$$63x^8 \cdot \frac{dx}{dt} \cdot y + 7x^9 \frac{dy}{dt} = 0$$

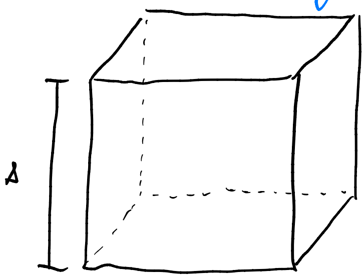
$$\frac{dy}{dt} = \frac{-63x^8 \cdot \frac{dx}{dt} \cdot y}{7x^9}$$

$7x^9y = 14$ @ $x=1 \Rightarrow 7(1)^9 y = 14$

$y = 2$

$$\left.\frac{dy}{dt}\right|_{x=1} = \frac{-63(1)^8 \cdot 2 \cdot 2}{7(1)^9} = -36$$

(2) Suppose all the edges of a cube are shrinking at a rate of 35 cm / sec. How fast is the surface area of the cube decreasing when the edge length is 11 cm?



$$A = 6s^2 \leftarrow \text{surface area}$$

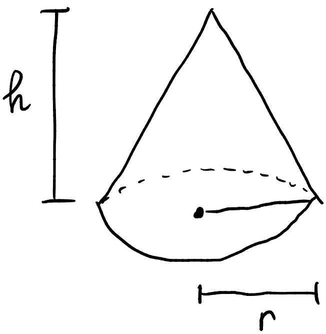
$$\frac{ds}{dt} = -35 \qquad \left.\frac{dA}{dt}\right|_{s=11} = ??$$

$$\frac{dA}{dt} = \frac{d}{dt}\left[6s^2\right] = 12s \cdot \frac{ds}{dt}$$

$$\left.\frac{dA}{dt}\right|_{s=11} = 12(11)(-35) = -4620 \text{ cm}^2/\text{sec}$$

The surface area is decreasing at a rate of $4620 \text{ cm}^2/\text{sec}$ when $s = 11$.

(4) Sand is being poured into a conical pile at a rate of $11\ cm^3/sec$. Length of the diameter of the pile is equal to the altitude of the pile. How fast is the altitude changing wrt. time t in seconds when the pile is 3 cm tall.



$V = \frac{1}{3}\pi r^2 h \leftarrow$ volume

$\frac{dV}{dt} = +11$

$2r = h$

$r = h/2$

$\left.\frac{dh}{dt}\right|_{h=3} = ??$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 \cdot h$

$V = \frac{\pi}{12} h^3$

$$\frac{dV}{dt} = \frac{d}{dt}\left[\frac{\pi}{12} h^3\right] = \frac{3\pi}{12} h^2 \cdot \frac{dh}{dt}$$

$$11 = \frac{\pi}{4} \cdot h^2 \cdot \frac{dh}{dt}$$

Solve for $\frac{dh}{dt}$

$$\frac{dh}{dt} = \frac{44}{\pi h^2}$$

$$\left.\frac{dh}{dt}\right|_{h=3} = \frac{44}{9\pi}\ cm/sec$$