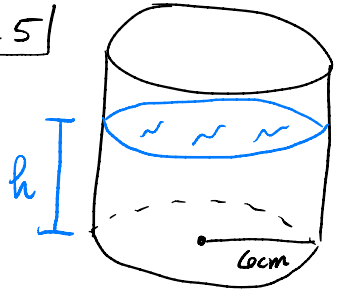
Lecture 15 : Related rates

HW 14 #5



water is drained at $26\ cm^3/sec$

$V = \pi r^2 h$ r: radius h: height $r = 6$

R.o.C. of the water level?

$\frac{dV}{dt} = -26$ $\frac{dh}{dt} = ?$

$$\frac{dV}{dt} = \frac{d}{dt}\left[\pi (6)^2 \cdot h\right]$$

$$\frac{dV}{dt} = 36\pi \frac{dh}{dt}$$

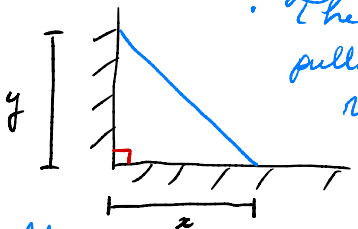
$$-26 = 36\pi \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{-26}{36\pi} \ cm/sec$$

The water level is decreasing at a rate of $\frac{26}{36\pi}$ cm/sec

e.g ①

- We have ladder of length $\sqrt{8}$ m
- The base of the ladder is being pulled away from the wall at a rate of $3/10$ m/s



How fast is the head of the ladder moving down when the base is 2 m from the wall.

$$x^2 + y^2 = (\sqrt{8})^2$$

$$x^2 + y^2 = 8 \qquad \frac{dx}{dt} = \frac{3}{10} \qquad \left.\frac{dy}{dt}\right|_{x=2} = ??$$

$$\frac{d}{dt}\left[x^2 + y^2\right] = \frac{d}{dt}[8]$$

$$2(x)\cdot\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Note if $x = 2$, then

$$(2)^2 + y^2 = 8$$
$$y^2 = 4$$
$$y = \pm 2$$

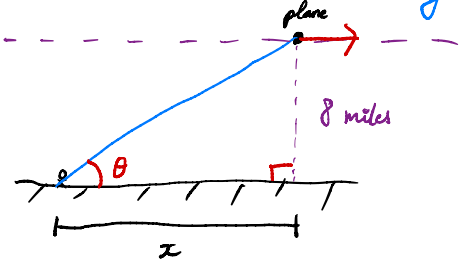
$$2\cdot 2\cdot\frac{3}{10} + 2\cdot 2\left.\frac{dy}{dt}\right|_{x=2} = 0$$

$$\left.\frac{dy}{dt}\right|_{x=2} = \frac{-3}{10} \text{ m/s}$$

the ladder is moving down at a rate of $3/10$ m/s.

(2) A plane is flying away from you at 350 mph at an altitude of 8 miles.

Find the rate at which the angle of elevation is decreasing when the angle is $\pi/6$.



$\frac{dx}{dt} = 350$ $\qquad \left.\frac{d\theta}{dt}\right|_{\theta=\frac{\pi}{6}} = ?$

$\tan\theta = \frac{8}{x}$

$\frac{d}{dt}[\tan\theta] = \frac{d}{dt}[8x^{-1}]$

$\sec^2(\theta) \cdot \frac{d\theta}{dt} = -8(x)^{-2} \cdot \frac{dx}{dt}$

if $\theta = \pi/6$, then $\tan\left(\frac{\pi}{6}\right) = \frac{8}{x}$

$\frac{1}{\sqrt{3}} = \frac{8}{x}$

$x = 8\sqrt{3}$

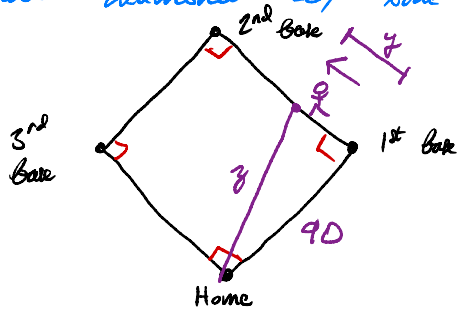
$\sec^2\left(\frac{\pi}{6}\right) \cdot \left.\frac{d\theta}{dt}\right|_{\theta=\frac{\pi}{6}} = -8(8\sqrt{3})^{-2} \cdot 350$

$\left.\frac{d\theta}{dt}\right|_{\theta=\frac{\pi}{6}} = -\frac{175}{16}$ rad/hr

θ is decreasing at a rate of $175/16$ rad/hr

(3) • Baseball diamond w/ side length 90 ft

Square →



player is running to 2nd Base at 12 ft/s

At what rate is the distance between the player and home increasing when the player is halfway between 1st and 2nd.

$$90^2 + y^2 = z^2 \qquad \frac{dy}{dt} = 12 \qquad \frac{dz}{dt}\Big|_{y=45} = ??$$

$$\frac{d}{dt}\left[90^2 + y^2\right] = \frac{d}{dt}\left[z^2\right]$$

$$2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$5\sqrt{405}$

$15\sqrt{45}$

[if $y = 45$, $\quad 90^2 + 45^2 = z^2$

$$z = \sqrt{10125}$$
$$= 45\sqrt{5}$$

$$2 \cdot 45 \cdot 12 = 2 \cdot 45\sqrt{5}\ \frac{dz}{dt}\Big|_{y=45}$$

$$\frac{dz}{dt}\Big|_{y=45} = \frac{12}{\sqrt{5}} \text{ feet/sec.}$$

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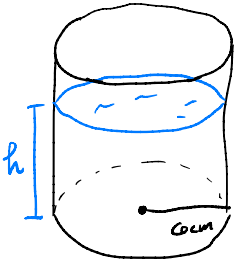


Water is drained out at a rate of $26\ cm^3/sec$

How fast does the water level of the tank drop?

r: radius
h: height

$V = \pi r^2 h$

$V = 36\pi h$

$\frac{dV}{dt} = -26$ $\quad \frac{dh}{dt} = ??$

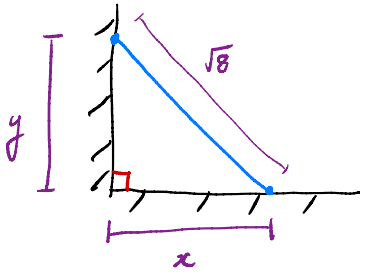
$$\frac{dV}{dt} = 36\pi \frac{dh}{dt}$$

$$-26 = 36\pi \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{26}{36\pi} \ cm/sec$$

The water level is decreasing at $\frac{26}{36\pi}$ cm/sec.

e.g. (1)



We have a ladder of length $\sqrt{8}$ m. The base of the ladder is being pulled away from the wall at a rate of $3/10$ m/s. How fast is the head of the ladder moving down when the base is 2 m from the wall.

$$x^2 + y^2 = (\sqrt{8})^2 = 8 \qquad \frac{dx}{dt} = \frac{3}{10} \qquad \frac{dy}{dt}\Big|_{x=2} = ??$$

$$\frac{d}{dt}\left[x^2 + y^2\right] = \frac{d}{dt}[8]$$

$$2(x)\cdot\frac{dx}{dt} + 2(y)\cdot\frac{dy}{dt} = 0$$

$x^2 = g(h(t))$

$g(u) = u^2$

$h(t) = x$

$x^2 = [h(t)]^2$

if $x = 2$, then

$(2)^2 + y^2 = 8$

$y^2 = 4$

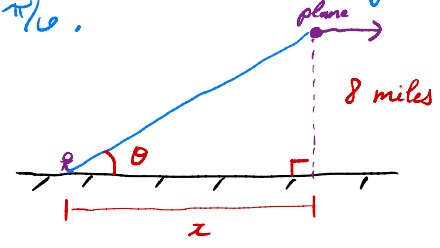
$y = \pm 2$

$$2\cdot 2\cdot\frac{3}{10} + 2\cdot 2\cdot\frac{dy}{dt}\Big|_{x=2} = 0$$

$$\frac{dy}{dt}\Big|_{x=2} = -\frac{3}{10} \text{ m/s}$$

the head of the ladder is moving down at $3/10$ m/s

(2) A plane is flying away from you at 350 mph at an altitude of 8 miles. Find the rate at which the angle of elevation is decreasing when the angle is $\pi/6$.



$\tan\theta = \frac{8}{x}$ $\qquad \frac{dx}{dt} = 350$ $\qquad \left.\frac{d\theta}{dt}\right|_{\theta=\frac{\pi}{6}} = ??$

$$\frac{d}{dt}[\tan\theta] = \frac{d}{dt}[8x^{-1}]$$

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if $\theta = \pi/6$, then

$$\tan\left(\frac{\pi}{6}\right) = \frac{8}{x}$$

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$$\sec^2\left(\frac{\pi}{6}\right) \left.\frac{d\theta}{dt}\right|_{\theta=\frac{\pi}{6}} = -8(8\sqrt{3})^{-2} \cdot 350$$

$$\left.\frac{d\theta}{dt}\right|_{\theta=\frac{\pi}{6}} = -\frac{175}{16}$$

θ decreases at $175/16$ rad/hr