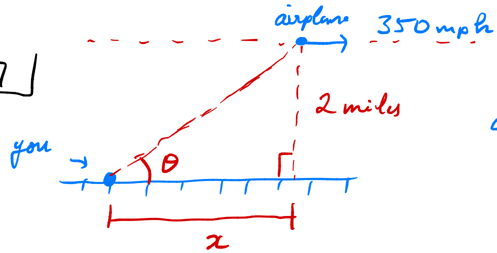
Lecture 16: Relative extrema and critical numbers

HW 15 #7



angle of elevation decreasing RoC when $\theta = \pi/6$?

$$\frac{dx}{dt} = 350 \qquad \left.\frac{d\theta}{dt}\right|_{\pi/6} = ?? \qquad \tan\theta = \frac{2}{x}$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = -2\,x^{-2}\,\frac{dx}{dt}$$

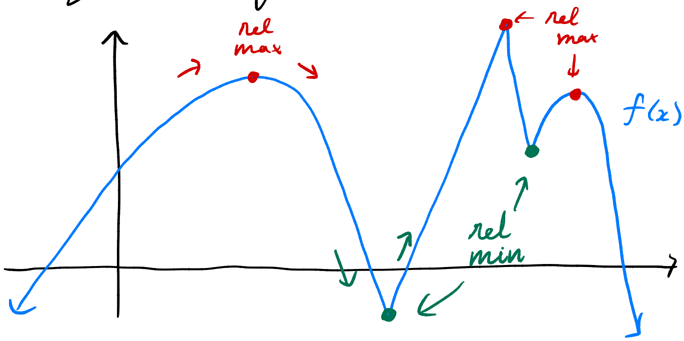
if $\theta = \pi/6$, then
$$\tan\frac{\pi}{6} = \frac{2}{x}$$
$$1/\sqrt{3} = 2/x$$
$$x = 2\sqrt{3}$$

$$\sec^2\left(\frac{\pi}{6}\right) \cdot \left.\frac{d\theta}{dt}\right|_{\theta=\pi/6} = -2\left(2\sqrt{3}\right)^{-2} \cdot 350 \checkmark$$

Relative extrema

a function goes ↗ then ↘ relative max

or function goes ↘ then ↗ relative min



Critical number

a value $x=c$ of a function $f(x)$ is a critical number of $f(x)$ if

1) $x=c$ is in the domain of f ($f(c)$ defined)

2) either $f'(c)=0$ OR $f'(c)$ DNE.

How to find relative extrema

Step 1: find all critical numbers

Step 2: Use the derivative near the critical numbers to test for sign changes.

e.g. find the critical numbers of $y=3x^2-6x$

$y'=6x-6$

$y'=0$	y' DNE
$6x-6=0$	X nothing here
$6x=6$	
$\underline{x=1}$	

critical number $x=1$

② $y = 9x^2 - \dfrac{9}{x^2}$ find all critical numbers

$y' = 18x + \dfrac{18}{x^3}$ note $x=0$ is not in the domain

$y' = 0$	y' DNE
$18x + \dfrac{18}{x^3} = 0$	$y' = 18x + \dfrac{18}{x^3}$
$18x = \dfrac{-18}{x^3}$	$= \dfrac{18x^4 + 18}{x^3}$
$18x^4 = -18$	good idea: check when denominator $= 0$.
$x^4 = -1$	$x^3 = 0$
no soln.	$x = 0$ ✗ not a critical number because not in domain

No critical numbers.

③ $y = \sqrt[3]{x}$ find the critical numbers.

$y = x^{1/3}$ $\qquad y' = \frac{1}{3} x^{-2/3} = \dfrac{1}{3\sqrt[3]{x^2}}$

$y' = 0$	y' DNE
$\sqrt[3]{x^2} \cdot \dfrac{1}{3\sqrt[3]{x^2}} = 0 \cdot \sqrt[3]{x^2}$	$\dfrac{1}{3\sqrt[3]{x^2}}$ DNE
$\dfrac{1}{3} = 0$ ✗	good idea: check when denominator $= 0$
no Soln	$3\sqrt[3]{x^2} = 0$ $\quad \boxed{x = 0}$

④ $y = 2\cos(8x) + 8x$ find all critical on $0 \leq x \leq \pi$

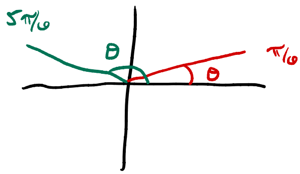
$$y' = -16\sin(8x) + 8$$

$\underline{y' \text{ DNE}}$ no critical number ✓

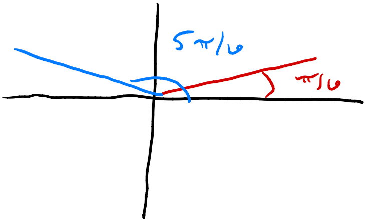
$\underline{y' = 0}$

$$-16\sin(8x) + 8 = 0$$
$$16\sin(8x) = 8$$
$$\sin(8x) = 1/2 \quad \text{on} \quad 0 \leq x \leq \pi$$

Recall $\sin\theta = 1/2$ $\rightsquigarrow$



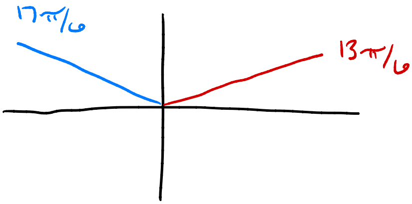
zero times around circle



$$8x = \frac{\pi}{6} \qquad 8x = \frac{5\pi}{6}$$

$$x = \frac{\pi}{6\cdot 8} = \frac{\pi}{48} \qquad x = \frac{5\pi}{6\cdot 8} = \frac{5\pi}{48}$$

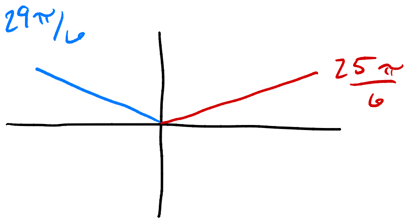
1 times around



$$8x = \frac{13\pi}{6} \qquad 8x = \frac{17\pi}{6}$$

$$x = \frac{13\pi}{48} \qquad x = \frac{17\pi}{48}$$

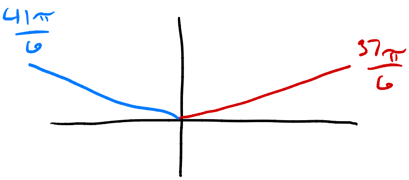
2 times around



$$8x = \frac{25\pi}{6} \qquad 8x = \frac{29\pi}{6}$$

$$x = \frac{25\pi}{48} \qquad x = \frac{29\pi}{48}$$

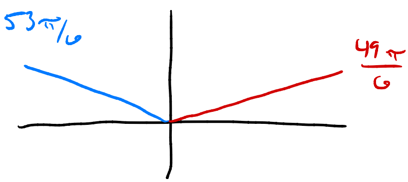
3 times around



$$8x = \frac{37\pi}{6} \qquad 8x = \frac{41\pi}{6}$$

$$x = \frac{37\pi}{48} \qquad x = \frac{41\pi}{48}$$

4 times around



$$8x = \frac{49\pi}{6} \qquad 8x = \frac{53\pi}{6}$$

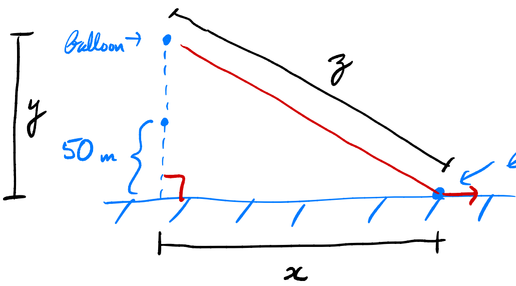
$$x = \frac{49\pi}{48} > \pi \qquad x = \frac{53\pi}{48} > \pi$$

too large!

Critical points: $\frac{\pi}{48}, \frac{5\pi}{48}, \frac{13\pi}{48}, \frac{17\pi}{48}, \frac{25\pi}{48}, \frac{29\pi}{48}, \frac{37\pi}{48}, \frac{41\pi}{48}$.

Lecture 16: Relative extrema and critical numbers.

HW 15 #81



- Balloon rising at 5 m/sec
- bike speed 10 m/sec

Bike: How fast is dist b/w bike and balloon incr. 10 secs later?

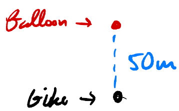
$$\frac{dy}{dt} = +5 \qquad \frac{dx}{dt} = +10 \qquad \left.\frac{dz}{dt}\right|_{t=10} = ??$$

$$x^2 + y^2 = z^2$$

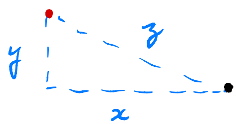
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

find x, y, z when $t = 10$.

$t = 0$



$t = 10$



$x = 100$ m since bike moving 10 m/sec

$y = 50\text{m} + 50\text{ m}$ since we started at 50m and rose at 5 m/sec

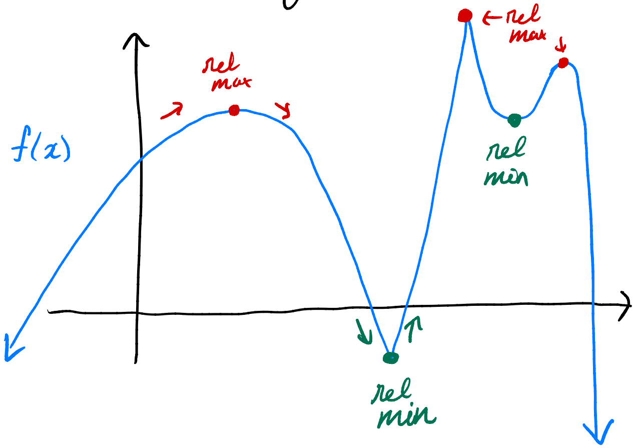
$$z = \sqrt{100^2 + 100^2} = 100\sqrt{2}$$

$$2(100)\cdot 10 + 2(100)5 = 2(100\sqrt{2})\left.\frac{dz}{dt}\right|_{t=10}$$

$$\left.\frac{dz}{dt}\right|_{t=10} = \frac{15}{\sqrt{2}} \text{ m/sec.}$$

Relative extrema

a function goes $\nearrow$ then $\searrow$ **relative max**
or goes $\searrow$ then $\nearrow$ **relative min**



$x=c$ is a **critical number** of a function $f(x)$ if
1) $x=c$ is in the domain of f ($f(c)$ exists)

2) $f'(c)=0$ or $f'(c)$ DNE.

How to find relative extrema:

Step 1: find all critical numbers

Step 2: determine if the sign of the derivative changes near the critical numbers.

e.g. $y = 3x^2 - 6x$ find all critical numbers.

$y' = 6x - 6$

$y' = 0$	y' DNE
$6x - 6 = 0$	Nothing!
$6x = 6$	
$x = 1$	$6x - 6$ always exists for real x.

critical number : $x = 1$

(2) $y = 9x^2 - \dfrac{9}{x^2}$ find all critical numbers.

$y' = 18x + \dfrac{18}{x^3}$ Note: y is not defined at $x = 0$

$y' = 0$	y' DNE
$18x + \dfrac{18}{x^3} = 0$	$y' = \dfrac{18x^4 + 18}{x^3}$
$18x = -\dfrac{18}{x^3}$	good idea: when is denominator $= 0$?
$18x^4 = -18$	$x^3 = 0$
$x^4 = -1$	✗ $x = 0$ ← not in domain of y.
no soln.	

No critical numbers

(3) $y = \dfrac{4x^2+8}{7x}$ find all critical numbers.

Note: $x=0$ is not in domain of y.

$$y' = \frac{8x(7x) - (4x^2+8)(7)}{(7x)^2}$$

$$= \frac{56x^2 - 28x^2 - 56}{49x^2}$$

$$y' = \frac{28x^2 - 56}{49x^2}$$

$y' = 0$	y' DNE
$\dfrac{28x^2-56}{49x^2} = 0$	denominator $= 0$
$28x^2 - 56 = 0$	$49x^2 = 0$
$x^2 = 2$	$\cancel{x} = 0$ not in domain
$x = \pm\sqrt{2}$	

Critical numbers: $x = \sqrt{2}$ $\quad x = -\sqrt{2}$.