Lecture 17: Increasing and decreasing functions; First derivative test

HW16 #10 $y = 5x^3 e^{4x+4}$ find critical numbers.

$$y' = (5x^3)' e^{4x+4} + 5x^3 \left(e^{4x+4}\right)'$$
$$= 15x^2 e^{4x+4} + 5x^3 \cdot e^{4x+4} \cdot 4$$
$$= 5x^2 e^{4x+4}(3 + 4x)$$

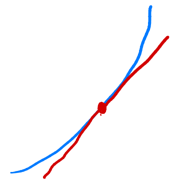
$y' = 0$	y' DNE
$5x^2 e^{4x+4}(3+4x) = 0$	nothing here
$5x^2 e^{4x+4} = 0$ or $3+4x=0$	
$5x^2 = 0$ $4x = -3$	
$x = 0$ $x = \frac{-3}{4}$	

Critical numbers: $x = \frac{-3}{4}$ and $x = 0$.

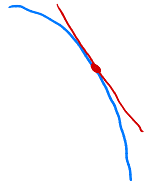
Recall an interval from a to b is all numbers x such that $a < x < b$.
We denote this by (a, b).

a could be $-\infty$ $(-\infty, b)$
b could be $+\infty$ $(a, +\infty)$

Increasing and decreasing functions



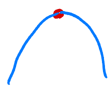
- If $f'(x) > 0$ on interval (a,b), then we say f is **increasing** on (a,b).



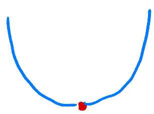
- If $f'(x) < 0$ on interval (a,b), then we say f is **decreasing** on (a,b).

A critical number $x=c$ of $f(x)$ is a ...

- **relative maximum** if the derivative of f switches from positive (inc) to negative (dec.) at $x=c$



- **relative minimum** if the derivative of f switches from negative to positive.



the first derivative test.

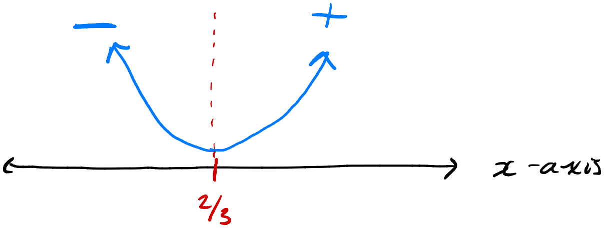
e.g. ① $y = 6x^2 - 8x$ find all relative extrema.

$y' = 12x - 8$

Critical numbers:

$y' = 0$	y' DNE
$12x - 8 = 0$	nothing here.
$12x = 8$	
$x = 2/3$	

Critical numbers: $x = 2/3$ $\qquad$ $y' = 12x - 8$



$y'(0) = 12(0) - 8 = -8$ $\qquad$ $y'(1) = 12(1) - 8 = 4$

So we see $x = 2/3$ is a relative min.

(2) $y = 5 + 6x - 8x^3$

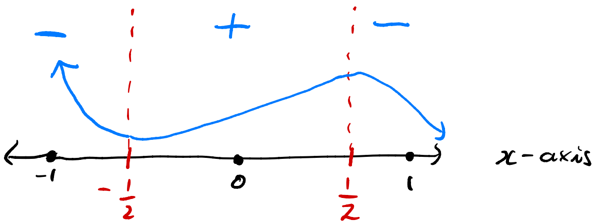
- find the relative extrema
- find where the function is increasing / decreasing.

$y' = 0 + 6 - 24x^2$

Critical numbers:

$y' = 0$	y' DNE
$6 - 24x^2 = 0$ $24x^2 = 6$ $x^2 = 1/4$ $x = \pm \frac{1}{2}$	Nothing here

Critical numbers $x = -\frac{1}{2}$ $\quad$ $x = \frac{1}{2}$



$y' = 6 - 24x^2$

$y'(-1) = 6 - 24(-1)^2 = 6 - 24 = -18$ negative

$y'(0) = 6 - 24(0)^2 = 6$ positive

$y'(1) = 6 - 24(1)^2 = 6 - 24 = -18$ negative

$x = -1/2$ relative min
$x = 1/2$ relative max.

y is increasing on the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$

y is decreasing on $\left(-\infty, -\frac{1}{2}\right)$ and $\left(\frac{1}{2}, +\infty\right)$

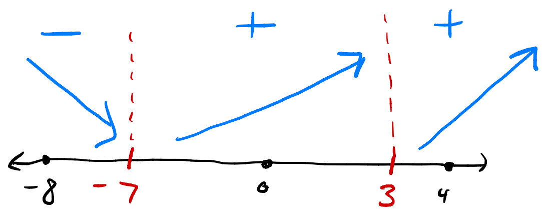
③ Let $f(x)$ be a function whose derivative is $(x-3)^2(x+7)$.
Find all rel. ext. and where f is inc./dec.

$$f'(x) = (x-3)^2(x+7)$$

Critical numbers.

$f'(x) = 0$	$f'(x)$ DNE
$(x-3)^2(x+7) = 0$	nothing here.
$x = 3$ or $x = -7$	

Critical numbers $x = 3$ $x = -7$



$$f'(-8) = (-8-3)^2(-8+7) = (-11)^2(-1) = -121$$

$$f'(0) = (0-3)^2(0+7) = (-3)^2 \cdot 7 = 63$$

$$f'(4) = (4-3)^2(4+7) = 1^2 \cdot 11 = 11$$

rel min $x = -7$

Lecture 17: Increasing and decreasing functions; First derivative test

HW 16 # 8 $y = \dfrac{2x^2+6}{2x}$ find the critical numbers.

$$y' = \frac{(4x)(2x) - (2x^2+6)(2)}{(2x)^2}$$

$$= \frac{8x^2 - 4x^2 - 12}{4x^2}$$

$$= \frac{4x^2-12}{4x^2} = \frac{x^2-3}{x^2}$$

y is not defined at $x=0$.

0 is not in the domain of y.

$y' = 0$	y' DNE
$\dfrac{x^2-3}{x^2} = 0$	set denominator $= 0$
$x^2 - 3 = 0$	$x^2 = 0$
$x^2 = 3$	$x = 0$ ← not in domain
$x = \pm\sqrt{3}$	

Critical numbers: $-\sqrt{3}$, $\sqrt{3}$

Recall an the interval from a to b includes all numbers x such that $a < x < b$ and we denote this by (a, b)

a may be $-\infty$ $(-\infty, b)$

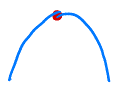
b may be $+\infty$ $(a, +\infty)$

Increasing and decreasing functions

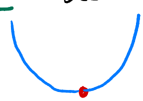
- If $f'(x) > 0$ on an interval (a,b), then we say f is increasing on (a,b).
- If $f'(x) < 0$ on an interval (a,b), then we say f is decreasing on (a,b).

A critical number $x=c$ of $f(x)$ is a ...

... relative maximum if $f'(x)$ switches from positive to negative at $x=c$



... relative minimum if $f'(x)$ switches from negative to positive at $x=c$



First derivative test.

e.g. ① $y = 6x^2 - 8x$

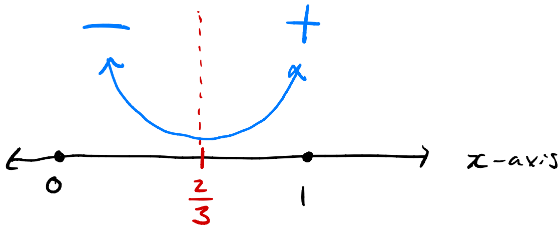
- find all relative extrema
- find where y inc./dec.

$y' = 12x - 8$

$y' = 0$	y' DNE
$12x - 8 = 0$ $12x = 8$ $x = 2/3$	nothing here.

Critical number : $x = 2/3$

$y' = 12x - 8$



$y'(0) = 12(0) - 8 = -8$

$y'(1) = 12(1) - 8 = 4$

We have $x = 2/3$ is a relative min.

y is increasing on the interval $\left(\frac{2}{3}, \infty\right)$

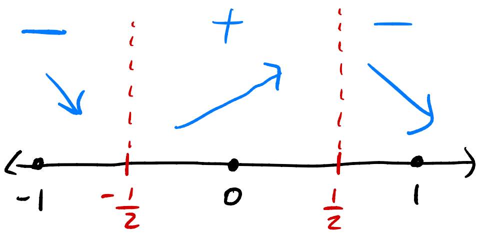
y is decreasing on the interval $\left(-\infty, \frac{2}{3}\right)$

(2) $y = 5 + 6x - 8x^3$ find the relative extrema.

$y' = 0 + 6 - 24x^2$

$y' = 0$	y' DNE
$6 - 24x^2 = 0$	nothing here
$24x^2 = 6$	
$x^2 = 1/4$	
$x = \pm \frac{1}{2}$	

Critical numbers: $x = -1/2$ $\quad x = 1/2$



$y'(-1) = 6 - 24(-1)^2 = 6 - 24 = -18$

$y'(0) = 6 - 24(0)^2 = 6$

$y'(1) = 6 - 24(1)^2 = -18$

$x = -1/2$ relative min

$x = 1/2$ relative max.

y is inc on $(-\frac{1}{2}, \frac{1}{2})$

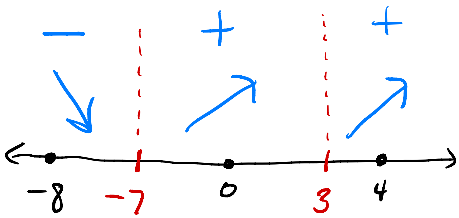
y is dec. on $(-\infty, -\frac{1}{2})$ and $(\frac{1}{2}, +\infty)$

③ Let $f(x)$ be a function whose derivative is $(x-3)^2(x+7)$. Find all rel. ext. of f.

$$f'(x) = (x-3)^2(x+7)$$

$f'(x) = 0$	$f'(x)$ DNE
$(x-3)^2(x+7) = 0$	nothing here.
$x = 3 \quad x = -7$	

Critical numbers: $x = -7, 3$.



$$f'(-8) = (-8-3)^2(-8+7) = (-11)^2(-1) = -121$$

$$f'(0) = (0-3)^2(0+7) = 9 \cdot 7 = 63$$

$$f'(4) = (4-3)^2(4+7) = 1^2 \cdot 11 = 11$$

$x = -7$ relative min

$x = 3$ neither.