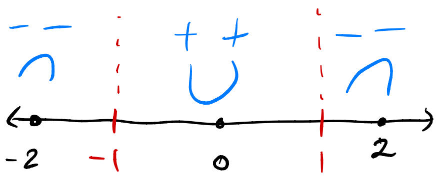
Lecture 19: Absolute extrema on an interval

HW 18 #71 $y = 6\ln(x^2+1)$ inflection pts?

$$y' = 6 \cdot 2x \cdot \frac{1}{x^2+1} = \frac{12x}{x^2+1}$$

$$y'' = \frac{12(x^2+1) - 12x(2x)}{(x^2+1)^2} = \frac{12 - 12x^2}{(x^2+1)^2}$$

$y'' = 0$	y'' DNE
$12 - 12x^2 = 0$	$(x^2+1)^2 = 0$
$12x^2 = 12$	$x^2 + 1 = 0$
$x^2 = 1$	$x^2 = -1$
$x = \pm 1$	no real soln.



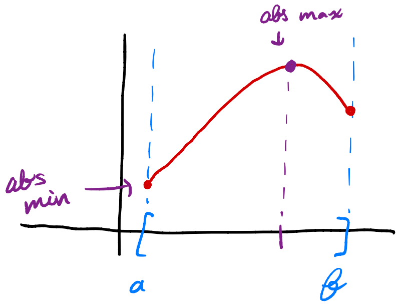
$y''(-2) < 0$ $\quad$ $y''(0) > 0$ $\quad$ $y''(2) < 0$

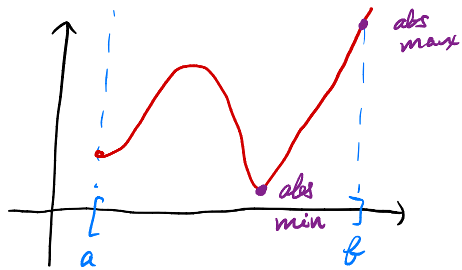
So $(-1, y(-1))$ and $(1, y(1))$ are inflect. pts.

Let f be a function defined on an interval I

1) $(c, f(c))$ is an absolute max of f on I if $f(c)$ is the largest value of f on I.

2) $(c, f(c))$ is an absolute min of f on I if $f(c)$ is the smallest value of f on I





How to find abs. extrema on $[a, b]$

1) find critical numbers
2) evaluate f at all critical numbers at the end points of the interval a and b

3) Determine the largest / smallest value.

$0 < x < 3$ $\quad (0, 3)$ $\qquad 0 \le x \le 3$

e.g. ① $y = 5xe^{-x} + 2$ find abs. extrema on $[0,3]$

$$y' = 5e^{-x} - 5xe^{-x} = 5e^{-x}(1-x)$$

$y' = 0$	y' DNE
$5e^{-x}(1-x) = 0$	nothing here.
$1 - x = 0$	
$x = 1$	

x	$y = 5xe^{-x} + 2$
0	$y(0) = 2$ abs min
1	$y(1) = 5e^{-1} + 2 \approx 3.84$ abs max
3	$y(3) = 15e^{-3} + 2 \approx 2.75$

$-1 < x \leq 5$

② $y = \dfrac{7x^2}{x+1}$ find abs min on $(-1, 5]$

← not defined at $x = -1$

$$y' = \frac{14x(x+1) - 7x^2(1)}{(x+1)^2} = \frac{7x^2 + 14x}{(x+1)^2} = \frac{7x(x+2)}{(x+1)^2}$$

$y' = 0$	y' DNE
$0 = \dfrac{7x(x+2)}{(x+1)^2}$	denominator $= 0$
$0 = 7x(x+2)$	$(x+1)^2 = 0$
$x = 0 \quad x = -2$	$x = -1$ ✗ not in domain

↑ not in $(-1, 5]$

Critical numbers: 0

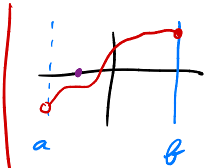
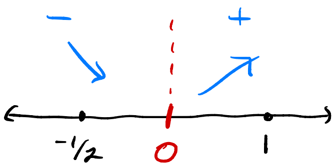


Fig 1

Need to check if $x = 0$ is a rel extrema first.



$$y'(-1/2) = \frac{7(-1/2)(-1/2 + 2)}{(-1/2 + 1)^2} < 0$$

$$y'(1) = \frac{7(1)(1+2)}{(1+1)^2} > 0$$

We see $x = 0$ is a rel min.

So Fig 1 cant happen.

x	$y(x) = \dfrac{7x^2}{x+1}$
0	$y(0) = 0$ ← abs min pt: $(0, 0)$.
5	$y(5) = 29.2$

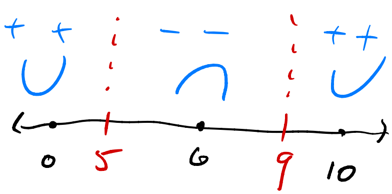
Lecture 19: Absolute extrema on an interval

HW 18 # 61 $f(x) = (x^2 - 18x + 79)e^x$ Inflection pts?

$$f'(x) = (2x - 18)e^x + (x^2 - 18x + 79)e^x$$
$$= e^x(x^2 - 16x + 61)$$

$$f''(x) = e^x(x^2 - 16x + 61) + e^x(2x - 16)$$
$$= e^x(x^2 - 14x + 45)$$

$f'' = 0$	f'' DNE
$e^x(x^2 - 14x + 45) = 0$	nothing here.
$x^2 - 14x + 45 = 0$	
$(x-5)(x-9) = 0$	
$x = 5, 9$	



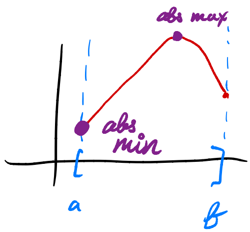
$f''(0) > 0$ $f''(6) < 0$ $f''(10) > 0$

Inf. pts: $(5, f(5))$ and $(9, f(9))$.

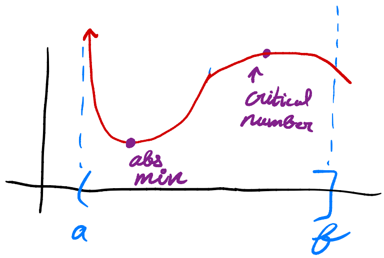
Let $f(x)$ be a function on an interval I

1) $(c, f(c))$ is an absolute max of f on I
   if $f(c)$ is the largest value f attains on I

2) $(c, f(c))$ is an abs min of f on I
   if $f(c)$ is the smallest value f attains on I



$a \leq x \leq b$



$a < x \leq b$

How to find abs. ext. on $[a, b]$

1) find crit. numbers
2) Evaluate f at critical numbers and a, b
3) Determine largest / smallest.

e.g. ① $y = 5xe^{-x} + 2$ on $[0,3]$ find abs. ext.

$$y' = 5e^{-x} - 5xe^{-x}$$
$$= 5e^{-x}(1-x)$$

$y' = 0$	y' DNE
$0 = 5e^{-x}(1-x)$	nothing here
$0 = 1 - x$	
$x = 1$	

Critical number: $x = 1$

x	$y(x) = 5xe^{-x} + 2$	
0	$y(0) = 2$	$\leftarrow$ abs min $(0, 2)$
1	$y(1) \approx 3.8$	$\leftarrow$ abs max $(1, 5e^{-1} + 2)$
3	$y(3) \approx 2.7$	

$-1 < x \leq 5$

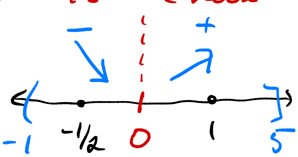
② $y = \dfrac{7x^2}{x+1}$ find abs min on $(-1, 5]$

$$y' = \frac{14x(x+1) - 7x^2(1)}{(x+1)^2}$$
$$= \frac{7x^2 + 14x}{(x+1)^2}$$
$$= \frac{7x(x+2)}{(x+1)^2}$$

$y' = 0$	y' DNE
$7x(x+2) = 0$	$(x+1)^2 = 0$
$x = 0, \cancel{-2}$ not in domain	$x = \cancel{-1}$ not in domain

Critical number: $x = 0$

Need to check if $x=0$ is rel. min.



$y'(-1/2) < 0$

$y'(1) > 0$

So $x = 0$ is rel min. ✓

x	$y(x)$
0	$y(0) = 0$
5	$y(5) \approx 29.2$

← abs min at $(0, 0)$.

③ $y = 1 - x^2 - 2x$ on $(-2, 0)$ find abs max.

1) find critical numbers
2) find rel max
3) choose largest value of above.

$y' = -2x - 2$

$y' = 0$	y' DNE
$-2x - 2 = 0$	nothing here
$x = -1$	

Use second derivative test for (2)

$y'' = -2$

$y''(-1) = -2 < 0$

2nd derivative test says $x = -1$ is a rel max

x	$y(x) = 1 - x^2 - 2x$
-1	$y(-1) = 2$

← abs max at $(-1, 2)$.