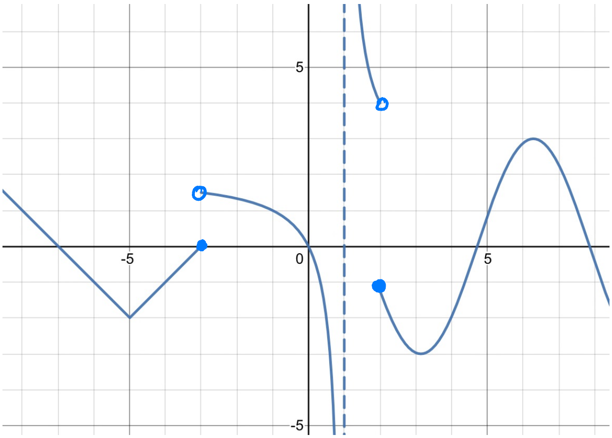
Lecture 2: Finding limits numerically, One-sided limits; Finding limits graphically

e.g. ① Let $f(x)$ be the function below.



$x = -3$

$\lim_{x \to -3^-} f = 0$

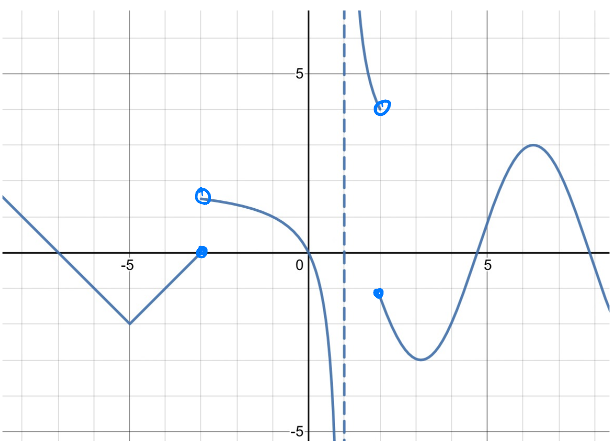
$\lim_{x \to -3^+} f = 1.5$

$\lim_{x \to -3} f = \text{DNE}$

$\lim_{x \to c} f(x)$ is the limit of f as x approaches c

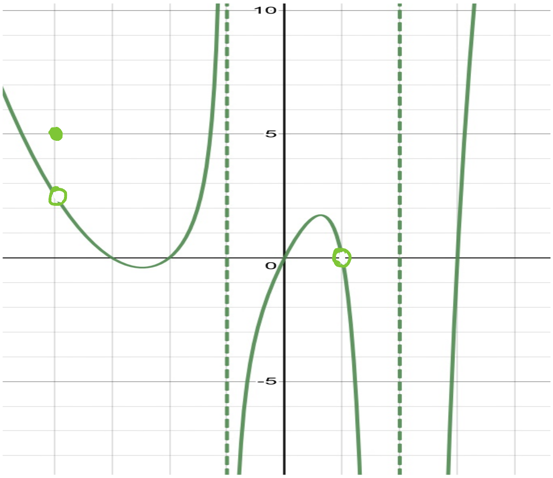
$\lim_{x \to c^+} f(x)$ " from the right

$\lim_{x \to c^-} f(x)$ " from the left



$x = -5$	$x = 1$
$\lim_{x \to -5^-} f = -2$	$\lim_{x \to 1^-} f = -\infty$
$\lim_{x \to -5^+} f = -2$	$\lim_{x \to 1^+} f = +\infty$
$\lim_{x \to -5} f = -2$	$\lim_{x \to 1} f = \text{DNE}$

② Let $g(x)$ be the function below



$x = -4$

$\lim_{x \to -4^-} g = 2.5$

$\lim_{x \to -4^+} g = 2.5$

$\lim_{x \to -4} g = 2.5$

$g(-4) = 5$

$x = 1$

$\lim_{x \to 1^-} g = 0$

$\lim_{x \to 1^+} g = 0$

$\lim_{x \to 1} g = 0$

$g(1) = \text{undef}$

Fact: $\lim_{x \to c} f(x)$ exists when

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$$

and we say $\lim_{x \to c} f(x) := \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$

③ $f(x) = 4x + 1$; $\lim_{x \to 2} f(x) = ?$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	8.6	8.96	8.996	9	9.004	9.04	9.4

$\lim_{x \to 2^-} f = 9$

$\lim_{x \to 2^+} f = 9$

$\lim_{x \to 2} f = 9$

(4) $g(x) = \dfrac{-5}{x-1}$; $\lim_{x \to 1} g(x)$

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$g(x)$	50	500	5000		-5000	-500	-50

$\lim_{x \to 1^-} g = +\infty \neq \lim_{x \to 1^+} g = -\infty$

$\lim_{x \to 1} g$ DNE

(5) $f(x) = \begin{cases} \cos x & x < 0 \\ x & x > 0 \end{cases}$ $\lim_{x \to 0} f(x)$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	.995	.9999	.9999		0.001	0.01	0.1

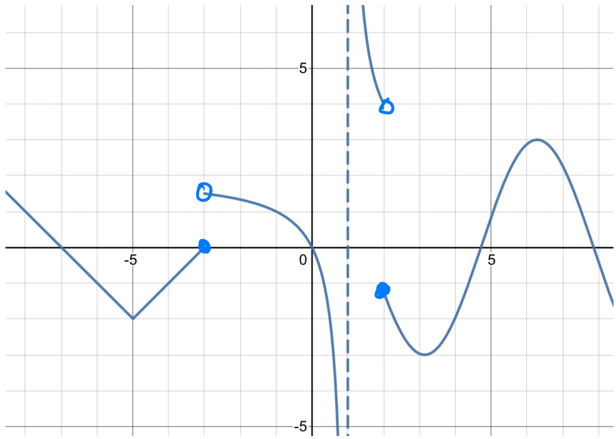
$\longrightarrow 1 \neq 0 \longleftarrow$

$\lim_{x \to 0^-} f = 1$ $\lim_{x \to 0^+} f = 0$

$\lim_{x \to 0} f$ DNE.

Lecture 2: Finding limits numerically, One-sided limits; Finding limits graphically

e.g. ① Let $f(x)$ be the function below

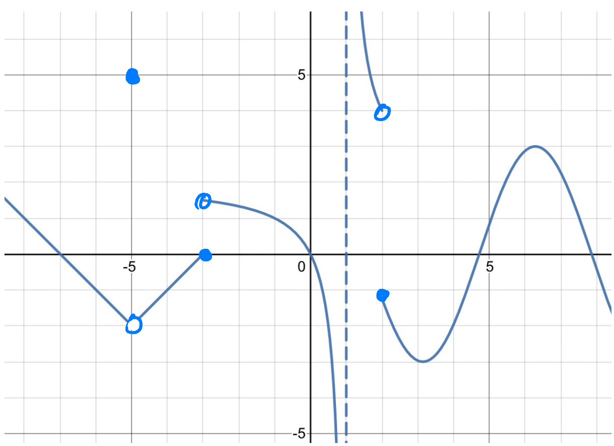


$x=-3$	$x=2$
$\lim_{x\to -3^-} f = 0$	$\lim_{x\to 2^-} f = 4$
$\lim_{x\to -3^+} f = 1.5$	$\lim_{x\to 2^+} f = -1$
$\lim_{x\to -3} f = \text{DNE}$	$\lim_{x\to 2} f = \text{DNE}$

$\lim_{x\to c} f(x)$ is the limit of f as x approaches c

$\lim_{x\to c^-} f(x)$ " from the left

$\lim_{x\to c^+} f(x)$ " from the right



$x=1$	$x=-5$
$\lim_{x\to 1^-} f(x) = -\infty$	$\lim_{x\to -5^-} f(x) = -2$
$\lim_{x\to 1^+} f(x) = +\infty$	$\lim_{x\to -5^+} f(x) = -2$
$\lim_{x\to 1} f(x) = \text{DNE}$	$\lim_{x\to -5} f(x) = -2$
	$f(-5) = 5$

Fact : $\lim\limits_{x\to c} f(x)$ exists, when

$$\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$$

and we say $\lim\limits_{x\to c} f(x) := \lim\limits_{x\to c^-} f(x) = \lim\limits_{x\to c^+} f(x)$

(2) $f(x) = \dfrac{\sin x}{x}$; $\lim\limits_{x\to 0} f(x)$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	$.99833$	$.99998$	$.99999$		$.99999$	$.99998$	$.99833$

$$\lim_{x\to 0^-} f(x) = 1 \qquad \lim_{x\to 0^+} f(x) = 1$$

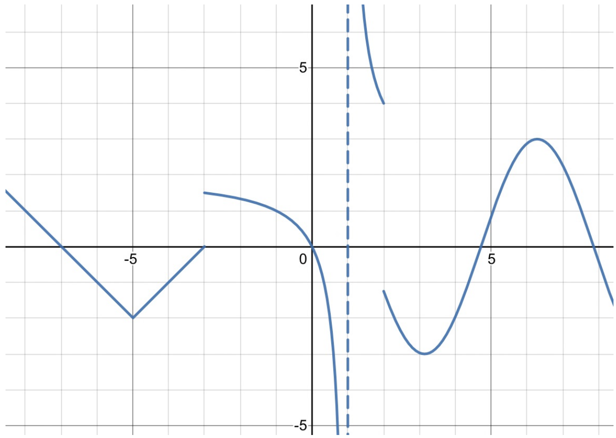
$$\lim_{x\to 0} f(x) = 1$$

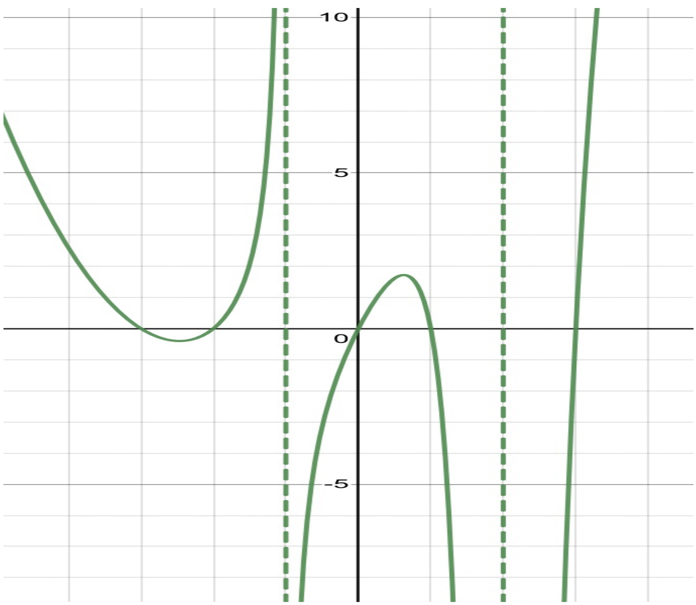
(3) $g(x) = \begin{cases} \cos x & x \leq 0 \\ x & x > 0 \end{cases}$; $\lim\limits_{x\to 0} g(x)$

x	-0.1	-0.01	0	0.01	0.1
$f(x)$	$.99500$	$.99995$		0.01	0.1

$$\lim_{x\to 0^-} g(x) = 1 \neq \lim_{x\to 0^+} g(x) = 0$$

$\Rightarrow \lim\limits_{x\to 0} g(x)$ DNE


5
-5
0
5
-5


10
5
0
-5