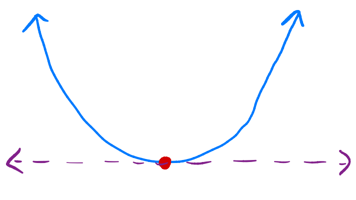
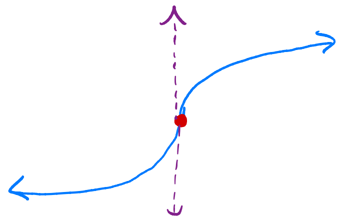
Lecture 20: Graphical interpretation of derivatives.

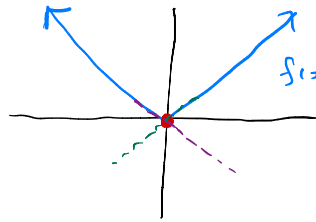
Critical numbers $f'(x) = 0$ or $f'(x)$ DNE



$f(x) = x^2$

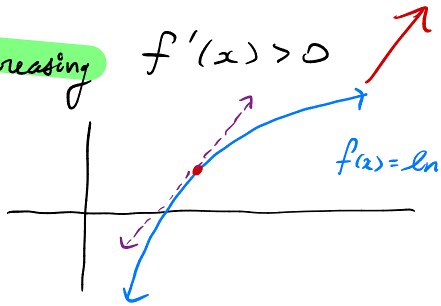


$f(x) = \sqrt[3]{x}$



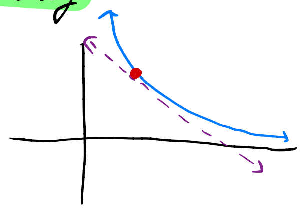
$f(x) = |x|$

Increasing $f'(x) > 0$ ↗



$f(x) = \ln(x)$

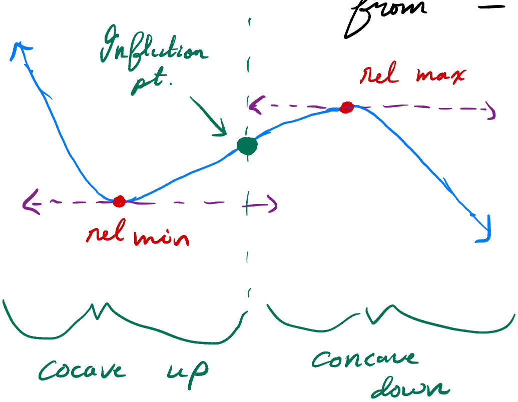
Decreasing $f'(x) < 0$ ↘



$f(x) = \frac{1}{x}$

Relative max : $(c, f(c))$ rel max if f' switches from $+$ to $-$. ↗ ↘

Relative min : $(c, f(c))$ is rel min if f' switches from $-$ to $+$ ↘ ↗



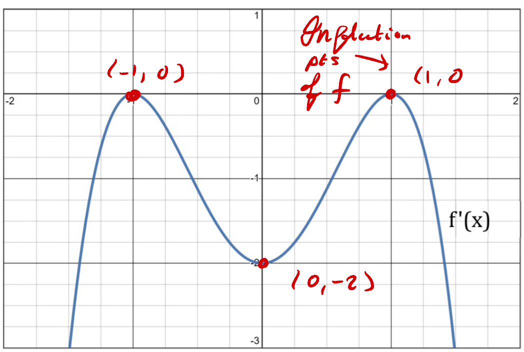
$f(x) = 2x - x^3$

Concave up: $f''(x) > 0$ ++ ◡

Concave down: $f''(x) < 0$ −− ⌒

Inflection pt: $(c, f(c))$ such that f'' switches signs.

① We are given the graph of $f'(x)$.



Critical numbers: $f' = 0$ or f' DNE.

$x = 1, -1$

Increasing: $f' > 0$ none

Decreasing: $f' < 0$

$(-\infty, \infty)$, $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

↑ for LON-CAPA

Concave up: $f'' > 0$

$(f')' > 0$ when is f' increasing?

concave up $(-\infty, -1)$, $(0, 1)$

Concave down: $f'' < 0$ ie when f' is decreasing

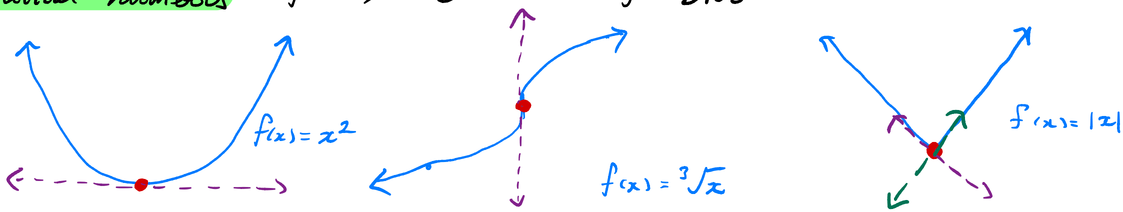
$(-1, 0)$ and $(1, \infty)$

Inflection: relative extrema for f'

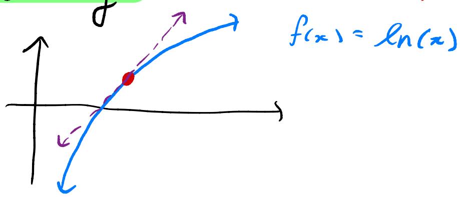
$(-1, 0)$ $(0, -2)$ and $(1, 0)$

Lecture 20: Graphical interpretation of derivatives.

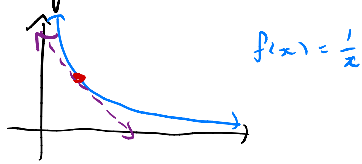
Critical numbers $f'(x) = 0$ or f' DNE



Increasing: $f'(x) > 0$ ↗



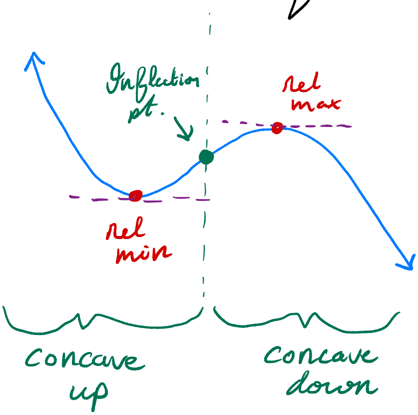
Decreasing $f'(x) < 0$ ↘



Relative max: $(c, f(c))$ is a rel max if f' switches from $+$ to $-$ ↗ ↘

Relative min: $(c, f(c))$ is a rel min if f' switches from $-$ to $+$ ↘ ↗

$f(x) = 2x - x^3$

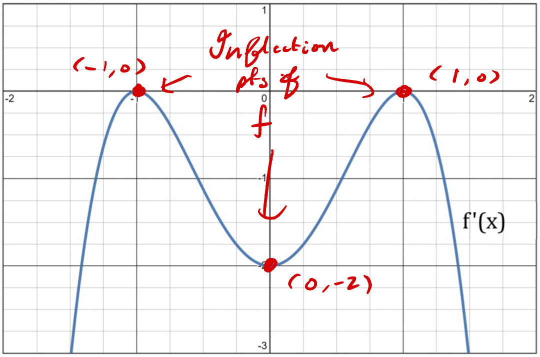


Concave up: $f''(x) > 0$ $++$ $\cup$

Concave down: $f''(x) < 0$ $--$ $\cap$

Inflection pt: $(c, f(c))$ such that f'' switches signs

① We are given the graph of $f'(x)$.



Critical numbers: $f'=0$ f' DNE
$x=-1$ and $x=1$

Increasing: $f'>0$ none

Decreasing: $f'<0$ $(-\infty,\infty)$

Relative extrema: none

Concave up: $f''>0$

$(f')'>0$ ie when is f' increasing?

$(-\infty,-1)$ $(0,1)$

Concave down: $f''<0$

$(f')'<0$ i.e., when is f' decreasing?

$(-1,0)$ and $(1,\infty)$

Inflection pt: $(c,f(c))$ where f'' switches signs

equivalent: $(c,f(c))$ rel. ext. for f'