Lecture 22: Summary of curve sketching.

① $y = \frac{x+3}{x-4}$ sketch the curve

Domain of y: what $\underline{\underline{\text{real}}}$ numbers can't be plugged into y?

domain of y = $(-\infty, 4)$ and $(4, \infty)$

x-intercept: Set $y = 0$. when function cross x-axis.

$$(x-4)\cdot 0 = \frac{x+3}{x-4}\cdot(x-4)$$

$$0 = x+3$$

$$x = -3$$

x-intercept at $(-3, 0)$

y-intercept Set $x=0$. when function cross y-axis

$$y = \frac{0+3}{0-4} = -\frac{3}{4}$$

y-intercept at $(0, -3/4)$

Behaviour at ± ∞

$$\lim_{x\to\infty}\frac{x+3}{x-4} = \lim_{x\to\infty}\frac{x}{x} = \lim_{x\to\infty}(1) = 1$$

$$\lim_{x\to-\infty}\frac{x+3}{x-4} = \lim_{x\to-\infty}\frac{x}{x} = \lim_{x\to\infty} 1 = 1$$

Horizontal Asym :

From behavior at ± ∞ we have a horiz asy. at $y = 1$.

Verticle asym. :

Assume no cancelation possible.

denominator = 0.

$$x - 4 = 0$$

$$x = 4$$

Verticle asy at $x = 4$.

Slant asym.

(must have deg top > deg of bot)

$x + 3$ ← deg is 1

$x - 4$ ← deg is 1

No slant asym.

Critical numbers

$y' = 0$ y' DNE check in domain

$$y = \frac{x+3}{x-4} \qquad y' = \frac{(1)(x-4) - (x+3)(1)}{(x-4)^2}$$

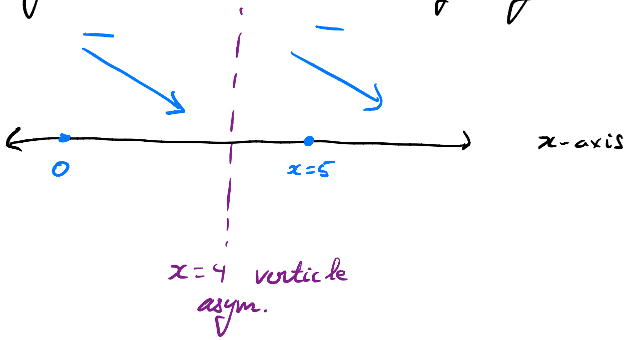
$$= \frac{-7}{(x-4)^2}$$

$y' = 0$	y' DNE
$\frac{-7}{(x-4)^2} = 0$	$0 = (x-4)^2$
$-7 = 0$	$x = \cancel{4}$ not in domain.
no soln.	

No critical numbers.

Increasing / decreasing?

Increasing $y' > 0$ decreasing $y' < 0$



$$y'(0) = \frac{-7}{(0-4)^2} = \frac{-7}{16} < 0$$

$$y'(5) = \frac{-7}{(5-4)^2} = \frac{-7}{1} < 0$$

decreasing on $(-\infty, 4)$ and $(4, \infty)$

increasing : none.

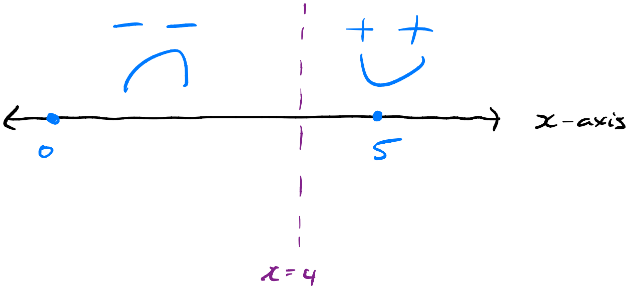
Rel. Extrema: No critical number, so no rel ext.

Concavity concave up $y'' > 0$ concave down $y'' < 0$.

$$y' = \frac{-7}{(x-4)^2} = -7(x-4)^{-2}$$

$$y'' = 14(x-4)^{-3}(1) = \frac{14}{(x-4)^3}$$

$y'' = 0$	y'' DNE
$0 = \dfrac{14}{(x-4)^3}$ $0 = 14$ no soln	$(x-4)^3 = 0$ $x = 4 \leftarrow$ not in domain not a possible inflection pt.



$y''(0) = \dfrac{14}{(0-4)^3} < 0$

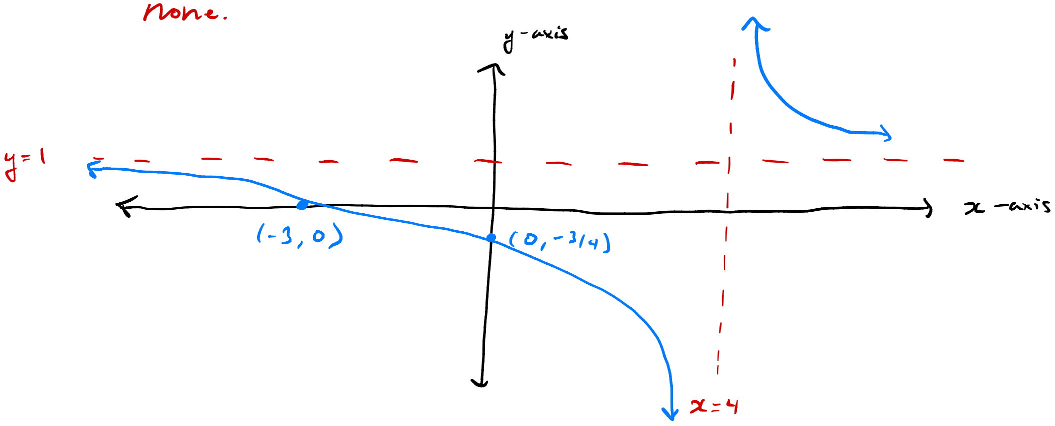
$y''(5) = \dfrac{14}{(5-4)^3} = 14 > 0$

Concave down : $(-\infty, 4)$

concave up : $(4, \infty)$

Inflection pts : pt in the domain st. $y'' = 0$ or y'' DNE

none.



Lecture 22 : A summary of curve sketching

① $y = \frac{x^2 - 3x + 9}{x - 3}$ sketch the function.

Domain : what points "break" the function?

domain of y is $(-\infty, 3)$ and $(3, +\infty)$

x-intercept Set $y = 0$. when the function crosses the x-axis.

$$0 = \frac{x^2 - 3x + 9}{x - 3}$$

$$0 = x^2 - 3x + 9$$

$$x = \frac{3 \pm \sqrt{9 - 4(1)(9)}}{2}$$

$$= \frac{3 \pm 3\sqrt{3}\sqrt{-1}}{2}$$ not a real number.

So no x-intercepts.

y-intercepts Set $x = 0$. when the function crosses the y-axis.

$$y = \frac{0^2 - 3(0) + 9}{(0) - 3} = \frac{9}{-3} = -3$$

y-intercept at $(0, -3)$.

Behaviour at ± ∞.

$$\lim_{x\to\infty}\frac{x^2-3x+9}{x-3}=\lim_{x\to\infty}\frac{x^2}{x}=\lim_{x\to\infty}x=+\infty$$

$$\lim_{x\to-\infty}\frac{x^2-3x+9}{x-3}=\lim_{x\to-\infty}\frac{x^2}{x}=\lim_{x\to-\infty}x=-\infty$$

$\frac{x(x^{-1})}{x}$

Horizontal Asym. none

Vertical Asym. make sure function has no possible cancellation.

denominator = 0.

$$x-3=0$$
$$x=3$$

Slant asym. deg of top > deg of bot

$$\frac{x^2-3x+9}{x-3}$$

← deg = 2

← deg = 1

$$\begin{array}{rl} & x \\ x-3\,\big) & \overline{x^2-3x+9} \\ & \underline{-(x^2-3x)} \\ & 0+0+9 \end{array}$$

#

$\frac{x^2}{x}=x$

x doesnt divide 9

$$\frac{x^2-3x+9}{x-3}=\underbrace{x}_{\text{slant}}+\frac{9}{x-3}$$

* note slant must be a line not a function w/ higher degree x's.

slant asym. at $y=x$.

Critical numbers: pts in domain st. $y' = 0$ or y' DNE

$$y = \frac{x^2 - 3x + 9}{x - 3} \qquad y' = \frac{(2x-3)(x-3) - (x^2-3x+9)(1)}{(x-3)^2}$$

$$= \frac{x(x-6)}{(x-3)^2}$$

$y' = 0$

$$0 = \frac{x(x-6)}{(x-3)^2}$$

$$0 = x(x-6)$$

$x = 0 \qquad x = 6$

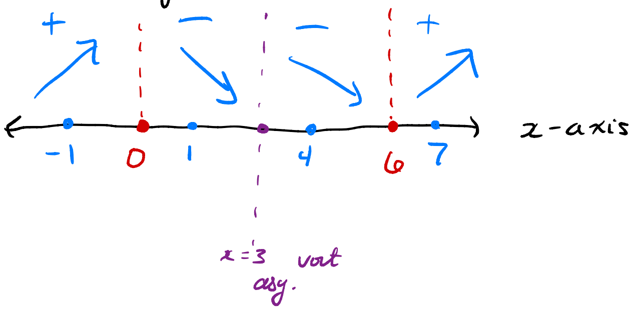
y' DNE

$$0 = (x-3)^2$$

$x = 3 \leftarrow$ not crit. #
b/c not in domain.

Crit #: $x = 0$ and $x = 6$

Increasing / decreasing



$$y'(-1) = \frac{(-1)(-1-6)}{(-1-3)^2} > 0 \qquad y'(4) = \frac{4(4-6)}{(4-3)^2} < 0$$

$$y'(1) = \frac{(1)(1-6)}{(1-3)^2} < 0 \qquad y'(7) = \frac{7(7-6)}{(7-3)^2} > 0$$

Increasing: $(-\infty, 0)$ and $(6, +\infty)$ Decreasing: $(0, 3)$ and $(3, 6)$

Relative Extrema

rel max @ $(0, y(0)) = (0, -3)$

rel min @ $(6, y(6)) = (6, 9)$

Concavity:

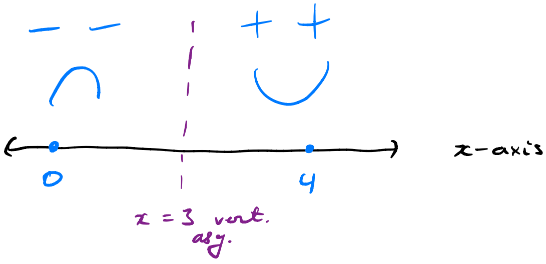
concave up $y'' > 0$

concave down $y'' < 0$.

$$y' = \frac{x^2 - 6x}{(x-3)^2} \qquad y'' = \frac{(2x-6)(x-3)^2 - (x^2-6x)\cdot 2(x-3)^1(1)}{(x-3)^4}$$

$$= \frac{18}{(x-3)^3}$$

$y'' = 0$	y'' DNE
$0 = \frac{18}{(x-3)^3}$	$0 = (x-3)^3$
$0 = 18$	$x = 3$ ← not in domain.
no soln.	



$$y''(0) = \frac{18}{(0-3)^3} < 0 \qquad y''(4) = \frac{18}{(4-3)^3} > 0$$

Concave up: $(3, \infty)$

Concave down: $(-\infty, 3)$.

Inflation pts none.

even though y'' switches sign at $x=3$ $x=3$ not in domain of y so it is not an infl. pt.