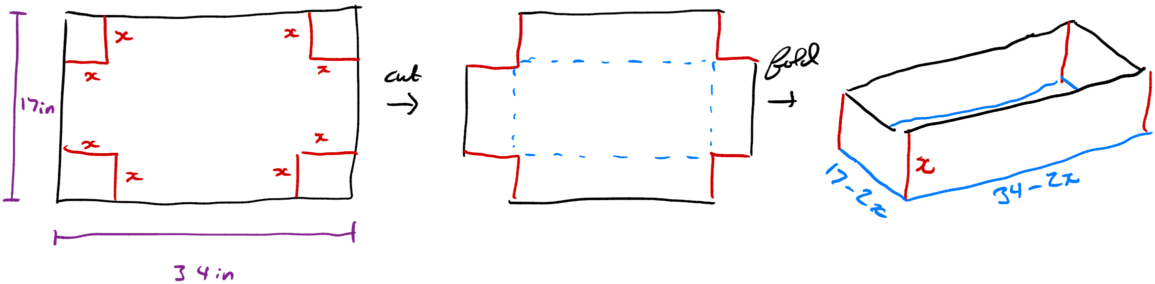
Lecture 23 : Optimization I

① A piece of cardboard 17 in × 34 in.
Cut a square of side length x into each corner
Fold cardboard into a box. Find the max volume.



Volume of box $V = x(17-2x)(34-2x)$

Objective function: The function we want to maximize (or minimize).

$$V = x(17-2x)(34-2x)$$

Constraint: $0 < x < 17/2$

Find the abs. max of V on interval $(0, 17/2)$

$$V = x(17-2x)(34-2x)$$
$$= 4x^3 - 102x^2 + 578x$$

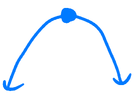
$$V' = 12x^2 - 204x + 578$$

looking in the domain of V $(0, 17/2)$

$V' = 0$	V' DNE
$0 = 12x^2 - 204x + 578$	nothing here.
$x \approx \cancel{13.4075}$ not in interval. or	
$x \approx 3.5925$	

$$V'' = 24x - 204$$

Second deriv. test $V''(3.5925) \approx 24(3.5925) - 204 < 0$



So $x = 3.5925$ is rel max.

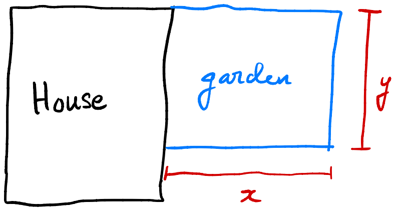
So $(3.5925, V(3.5925))$ is the abs max

The max volume is

$$V(3.5925) \approx (3.5925)(17 - 2\cdot 3.5925)(34 - 2\cdot 3.5925)$$

$$\approx 945.5073 \text{ in}^3$$

② You have 100 ft of fence to make a rectangular garden along side the wall of your house. What is the largest possible area of your garden?



Obj: $A = xy$

Const: $100 = 2x + y$

$y = 100 - 2x$

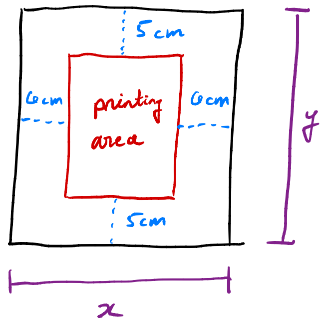
$y > 0$
$100 - 2x > 0$
$100 > 2x$
$x < 50$

Obj: $A = x(100 - 2x)$

Const: $0 < x < 50$

Find abs. max of $A = x(100 - 2x)$ on interval $(0, 50)$.

③ You are designing a poster with area 900 cm^2



Choose x and y so that the printing area is maximized.

Obj: $A = (x-12)(y-10)$

Const: $900 = xy$; $x > 12$ $y > 10$

$$y = \frac{900}{x}$$

$y > 10$

$\frac{900}{x} > 10$

$\frac{1}{x} > \frac{1}{90}$

$x < 90$

Obj: $A = (x-12)\left(\frac{900}{x} - 10\right)$

Const: $12 < x < 90$

Find the abs. max of $A = (x-12)\left(\frac{900}{x} - 10\right)$ on the interval $(12, 90)$.

$\rightsquigarrow$ this tells us x

Use $y = \frac{900}{x}$ to find y.

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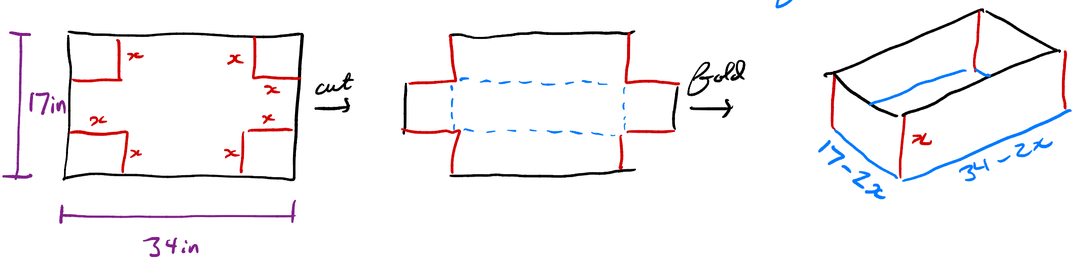
Fold up cardboard into a box.
What is the maximum volume of the box?



Objective function: The function we want to maximize (or minimize).

$$V = x(17-2x)(34-2x)$$

Constraint: $0 < x < 17/2$

Find the abs. max. of $V = x(17-2x)(34-2x)$ on the interval $(0, 17/2)$.

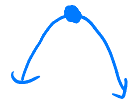
$$V = 4x^3 - 102x^2 + 578x$$
$$V' = 12x^2 - 204x + 578$$

look in domain of V: $(0, 17/2)$

$V' = 0$	V' DNE
$0 = 12x^2 - 204x + 578$ ~~$x \approx 13.4075$~~ not in domain or $x \approx 3.5925$	nothing here.

$V'' = 24x - 204$

$V''(3.5925) = 24(3.5925) - 204 < 0$

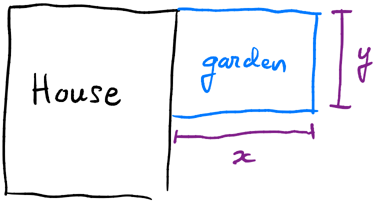


Second deriv $\Rightarrow$ $x = 3.5925$ is a rel max

So $(3.5925, V(3.5925))$ is the abs. max.

$V(3.5925) = 945.5073 \text{ in}^3$

② You have 100 ft of fence to make a rectangular garden along side the wall of your house. What is the largest possible area of your garden?



Obj : $A = xy$

Const : $100 = 2x + y$; $x > 0$; $y > 0$

$y = 100 - 2x$

$y > 0$

$100 - 2x > 0$

$-2x > -100$

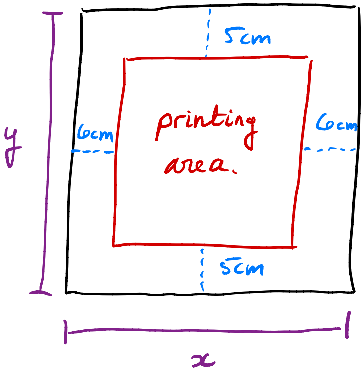
$x < 50$

Obj : $A = x(100 - 2x)$

Const : $0 < x < 50$

Find the abs. max of $A = x(100 - 2x)$ on the interval $(0, 50)$.

③ You are designing a poster with area $900\ cm^2$



Find the largest printing area.

Obj : $A = (x-12)(y-10)$

Const : $900 = xy$; $x > 12$; $y > 10$

$$y = \frac{900}{x}$$

$y > 10$

$\frac{900}{x} > 10$

$\frac{1}{x} > \frac{1}{90}$

$x < 90$

Obj : $A = (x-12)\left(\frac{900}{x} - 10\right)$

Const : $12 < x < 90$

Find the abs. max of $A = (x-12)\left(\frac{900}{x} - 10\right)$ on the interval $(12, 90)$.