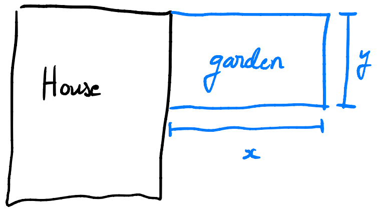
Lecture 24: Optimization II

HW 23 # 4 3L feet of fence to make a rect. garden along the side of our house where L is a positive constant. What is largest possible area?



Obj : $A = xy$

Const : $3L = 2x + y$; $x > 0$; $y > 0$

$y = 3L - 2x$

$3L - 2x > 0$

$-2x > -3L$

$x < \frac{3L}{2}$

Obj : $A = x(3L - 2x)$

Const : $0 < x < \frac{3L}{2}$

Find abs. max of $A = x(3L - 2x)$ on $\left(0, \frac{3L}{2}\right)$

$A = 3Lx - 2x^2$

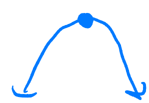
$A' = 3L - 4x$

$0 = 3L - 4x$

$4x = 3L$

$x = \frac{3L}{4}$ ← only crit # and its in the interval.

$A'' = -4$ $\qquad$ $A''\left(\frac{3L}{4}\right) = -4 < 0$



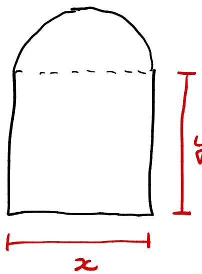
rel max !

Only rel max on $\left(0, \frac{3L}{2}\right)$ so its abs. max.

$$A\left(\frac{3L}{4}\right) = 3L\left(\frac{3L}{4}\right) - 2\left(\frac{3L}{4}\right)^2 = \frac{9L^2}{4} - \frac{18L^2}{16}$$

$$= \frac{18L^2}{16} = \frac{9L^2}{8} \text{ ft}^2.$$

① A window has perimeter 24 ft.
Find dim of window to let the most light in.



Obj: $A = (\text{Area of rectangle}) + (\text{Area of circle})$
$= xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

Const: $24 = x + y + \frac{1}{2}\left(2\pi \cdot \frac{x}{2}\right) + y$
$24 = \frac{\pi}{2}x + x + 2y$; $x > 0$; $y > 0$

$2y = 24 - \frac{\pi}{2}x - x$
$y = 12 - \frac{\pi}{4}x - \frac{1}{2}x$

$y > 0$
$12 - \frac{\pi}{4}x - \frac{1}{2}x > 0$
$12 - \frac{2+\pi}{4}x > 0$
$-\frac{2+\pi}{4}x > -12$
$x < \frac{48}{2+\pi}$

Obj: $A = x\left(12 - \frac{\pi}{4}x - \frac{1}{2}x\right) + \frac{\pi}{8}x^2$
$= 12x - \frac{\pi}{4}x^2 - \frac{1}{2}x^2 + \frac{\pi}{8}x^2$
$= 12x + \frac{\pi - 2\pi - 4}{8}x^2$

Const: $0 < x < \frac{48}{2+\pi}$

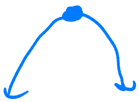
$A' = 12 + \frac{\pi - 2\pi - 4}{8} \cdot 2x$

$0 = 12 - \frac{\pi + 4}{4}x$

$x = \frac{48}{4+\pi}$ ← only crit #
Is it in $\left(0, \frac{48}{2+\pi}\right)$? Yes!

Does $x = \dfrac{48}{4+\pi}$ correspond to rel max.

$$A'' = -\frac{\pi+4}{4} \qquad A''\left(\frac{48}{4+\pi}\right) = -\frac{\pi+4}{4} < 0$$

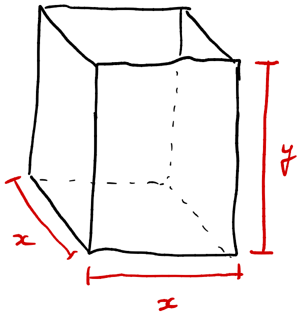


So $x = \dfrac{48}{4+\pi}$ is rel max

So its the abs max.

$$y = 12 - \frac{1}{2}\left(\frac{48}{4+\pi}\right) - \frac{\pi}{4}\left(\frac{48}{4+\pi}\right)$$

(2) A box with a square base, no top, volume = 6912 in^3. Find dim. of box that require least ammount of material.



Obj: $S = x^2 + 4xy$

Const: $6912 = x^2 y$; $x > 0$; $y > 0$

$$y = \frac{6912}{x^2}$$

$y > 0$

$\frac{6912}{x^2} > 0$

$\frac{1}{x^2} > 0$

always true!

Obj: $S = x^2 + 4x\left(\frac{6912}{x^2}\right) = x^2 + \frac{4 \cdot 6912}{x}$

Const: $0 < x < +\infty$

Goal: Find the abs min of $S = x^2 + \frac{4 \cdot 6912}{x}$ on the interval $(0, \infty)$.

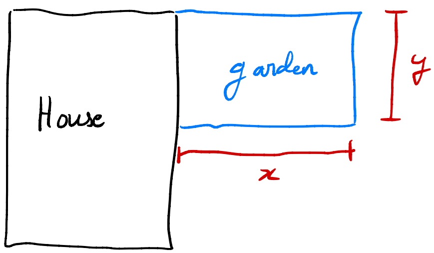
$$S' = 2x - \frac{4 \cdot 6912}{x^2} = \frac{2x^3 - 4 \cdot 6912}{x^2}$$

Crit #'s

$S' = 0$ or	S' DNE
$0 = 2x^3 - 4 \cdot 6912$	$x^2 = 0$
$x = 24$	$x = 0$ ← not in domain
↑ in domain	

Lecture 24: Optimization II

HW23 #4 3L feet of fence to make a rectangular garden alongside our house. What is largest possible area of the garden? Here L is a positive constant.



Obj: $A = xy$

Const: $3L = 2x + y$; $x > 0$; $y > 0$

$y = 3L - 2x$

$3L - 2x > 0$

$-2x > -3L$

$x < \frac{3L}{2}$

Obj: $A = x(3L - 2x) = 3Lx - 2x^2$

Const: $0 < x < \frac{3L}{2}$

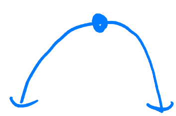
Crit #'s: $A' = 3L - 4x$

$0 = 3L - 4x$

$x = \frac{3L}{4}$ is in $\left(0, \frac{3L}{2}\right)$ ✓

Is $x = \frac{3L}{4}$ rel max?

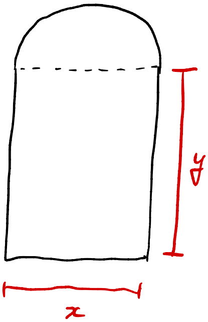
$A'' = -4$ $\quad A''\left(\frac{3L}{4}\right) = -4 < 0$



So $x = \frac{3L}{4}$ is rel max, in fact abs max

$$A\left(\frac{3L}{4}\right) = 3L\left(\frac{3L}{4}\right) - 2\left(\frac{3L}{4}\right)^2 = \frac{9L^2}{4} - \frac{9L^2}{8} = \frac{9L^2}{8} \text{ ft}^2$$

① A window has perimeter 24 ft. Find the dim of window that allow the most light.



Obj: $A = (\text{Area of } \square) + (\text{Area of } \frown)$
$= xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

Const: $24 = (\text{perm. } \sqcup) + (\text{perm } \frown)$
$x + 2y + \frac{1}{2}\pi x$;
$x > 0$; $y > 0$

$$2y = 24 - x - \frac{\pi}{2}x$$
$$y = 12 - \frac{2+\pi}{4}x$$

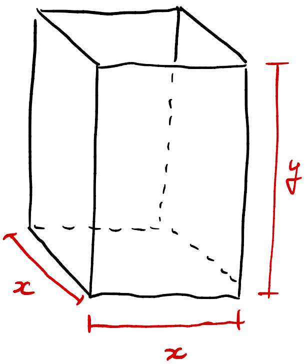
Obj: $A = x\left(12 - \frac{2+\pi}{4}x\right) + \frac{\pi}{8}x^2$
$= 12x + \frac{-4-2\pi+\pi}{8}x^2$
$= 12x - \frac{4+\pi}{8}x^2$

$y > 0$
$12 - \frac{2+\pi}{4}x > 0$
$x < \frac{48}{2+\pi}$

Const: $0 < x < \frac{48}{2+\pi}$

Goal: Find the abs max of $A = 12x - \frac{4+\pi}{8}x^2$ on the interval $\left(0, \frac{48}{2+\pi}\right)$.

(2) A box with a square base, no top, volume of $6912\,in^3$. Find dim of box that require the least ammount of material.



Obj: $S = x^2 + 4xy$

Const: $6912 = x^2 y$; $x > 0$; $y > 0$

$$y = \frac{6912}{x^2}$$

$y > 0$

$\frac{6912}{x^2} > 0$

$\frac{1}{x^2} > 0$

always true for $x > 0$!

Obj: $S = x^2 + 4x\left(\frac{6912}{x^2}\right) = x^2 + \frac{4 \cdot 6912}{x}$

Const: $0 < x < +\infty$

Goal: Find abs min of $S = x^2 + \frac{4 \cdot 6912}{x}$ on $(0, \infty)$.

$$S' = 2x - \frac{4 \cdot 6912}{x^2} = \frac{2x^3 - 4 \cdot 6912}{x^2}$$

Crit #'s:

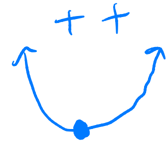
$S' = 0$ or	S' DNE
$0 = 2x^3 - 4 \cdot 6912$	$0 = x^2$
$x = 24$	$x = \cancel{0}$ not in $(0, \infty)$

Is $x = 24$ the x-value of a rel min?

$$S' = 2x - \frac{4 \cdot 6912}{x^2}$$

$$S'' = 2 + \frac{2 \cdot 4 \cdot 6912}{x^3}$$

$$S''(24) = 2 + \frac{2 \cdot 4 \cdot 6912}{(24)^3} > 0$$



So $x = 24$ is x-value of rel min

So $x = 24$ is x-value of abs min

$$\underbrace{x = 24 \text{ in} \qquad y = \frac{6912}{(24)^2} \text{ in}}$$

dim of box that minimize the material needed.