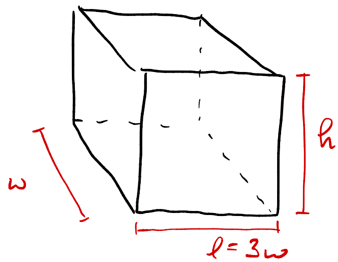
Lecture 25: Optimization III

① Construct a box w/ volume $35\ ft^3$ using metal and wood.

metal cost : $\$14 / ft^2$
wood cost : $\$8 / ft^2$

Put wood on the sides and metal on top/bot.
Length of base is 3 times the width of base
Find dim. of box that minimize cost of construction.



$$\text{Obj}: \ C = 14(3w^2 + 3w^2) + 8(3wh + wh + 3wh + wh)$$
$$= 14 \cdot 6w^2 + 8 \cdot 8wh$$
$$= 84w^2 + 64wh$$

$$\text{Const}: \ 35 = 3w \cdot w \cdot h = 3w^2 h; \quad w > 0; \ h > 0$$
$$h = \frac{35}{3w^2}$$

$$\frac{35}{3w^2} > 0$$
$$\frac{1}{w^2} > 0$$
$$1 > 0$$

$$\text{Obj}: \ C = 84w^2 + 64w\left(\frac{35}{3w^2}\right)$$
$$= 84w^2 + \frac{2240}{3w}$$

$$\text{Const}: \ 0 < w < \infty$$

Find the abs. min of $C = 84w^2 + \frac{2240}{3w}$

on the interval $(0, +\infty)$.

(2) If a company sells a product at p dollars per unit they will sell

$$q = 2800 - 100p$$

units. Each unit costs \$3 to make.

a) What price should the company charge to maximize revenue?

Revenue = (price per unit)(# of units sold)

Obj: $R = p(2800 - 100p)$
$= 2800p - 100p^2$

$p \geq 28$, then
$q = 2800 - 100p \leq 0$

Const: $0 < p < 28$

Find the abs max of $R = 2800p - 100p^2$ on $(0, 28)$.

b) What should p be to maximize profit?

Profit = Revenue − (cost per unit)(# of units sold)

Obj: $P = 2800p - 100p^2 - 3(2800 - 100p)$
$= -100p^2 + 3100p - 8400$

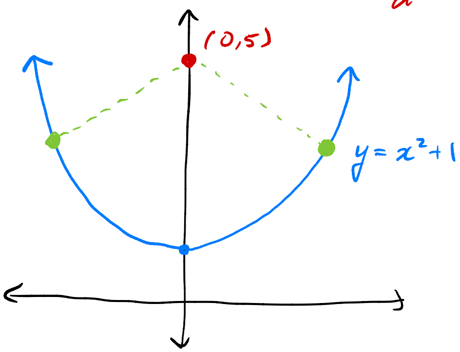
Const: $0 < p < 28$

Find abs. max of P on interval $(0, 28)$.

③ Find the points on the curve $y = x^2 + 1$ closest to the point $(0,5)$.

Recall: distance between (x, y) and (x_0, y_0) is

$$d^2 = (x - x_0)^2 + (y - y_0)^2$$



Obj: $D = (x-0)^2 + (y-5)^2$

Const: $y = x^2 + 1$

$x^2 = y - 1$

Obj: $D = y - 1 + (y-5)^2 = y^2 - 9y + 24$

Const: $1 \leq y < +\infty$

Find the abs min of $D = y^2 - 9y + 24$ on $[1, \infty)$.

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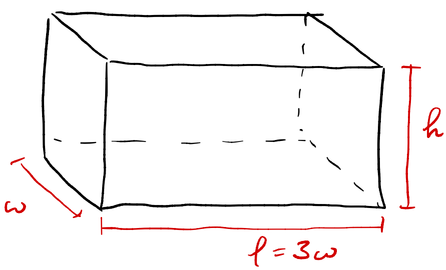

Obj: $C = 14(3w\cdot w + 3w\cdot w) + 8(3wh + wh + 3wh + wh)$

$= 14\cdot 6w^2 + 8\cdot 8wh$

$= 84w^2 + 64wh$

Const: $35 = (3w)(w)(h) = 3w^2h$; $w > 0$; $h > 0$

$h = 35/3w^2$

$\frac{35}{3w^2} > 0$

$\frac{1}{w^2} > 0$

always true for $w > 0$

Obj: $C = 84w^2 + 64w\left(\frac{35}{3w^2}\right)$

$= 84w^2 + \frac{2240}{3w}$

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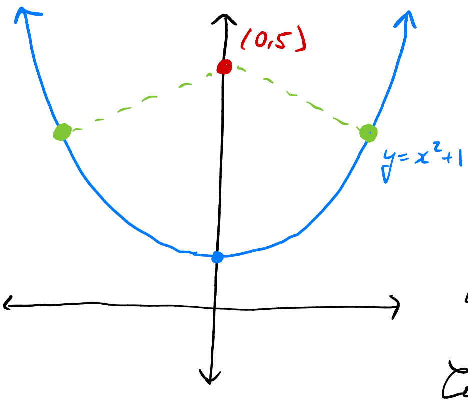


Recall: the distance between two points (x,y) and (x_0,y_0) is

$$d^2 = (x-x_0)^2 + (y-y_0)^2$$

Obj: $D = (x-0)^2 + (y-5)^2$

$D = x^2 + (y-5)^2$

Const: $y = x^2+1$

$x^2 = y-1$

Obj: $D = y-1 + (y-5)^2$

$= y^2 - 9y + 24$

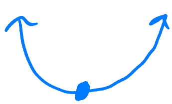
Const: $1 \leq y < \infty$

Find the abs. min of $D = y^2 - 9y + 24$ on the interval $[1, \infty)$.

$D' = 2y - 9$

Crit #s	$D' = 0$	D' DNE
	$0 = 2y - 9$ $y = 9/2$	nothing here.

Is $y = 9/2$ a rel min? $D'' = 2$ $\quad D''(9/2) = 2 > 0$



Yes!

y	D
1	$D(1) = 16$
$9/2$	$D(9/2) = 3.75$

$\leftarrow$ $(9/2, 3.75)$ is abs min of D.

$$x^2 = y - 1 \quad = \quad x^2 = 9/2 - 1$$
$$x = \pm\sqrt{9/2 - 1}$$

$$\left(-\sqrt{9/2 - 1},\ 9/2\right) \quad \text{and} \quad \left(\sqrt{9/2 - 1},\ 9/2\right).$$