Lecture 27: Antiderivatives and indefinite integration II

HW 26 #8

$$\int \frac{1+8xe^x}{x}\,dx$$

$$= \int \frac{1}{x} + \frac{8xe^x}{x}\,dx$$

$$= \int \frac{1}{x}\,dx + 8\int \frac{xe^x}{x}\,dx$$

$$= \int \frac{1}{x}\,dx + 8\int e^x\,dx$$

$$= \ln|x| + 8e^x + C$$

LON-CAPA $\downarrow$ abs(x)

Recall: A function $F(x)$ is an **antiderivative** of $f(x)$ if $F'(x) = f(x)$

We define the **indefinite integral** of $f(x)$ as

$$\int f(x)\,dx := F(x) + C$$

① Let $\underbrace{f''(x) = 3e^x + 4}_{\text{ODE ordinary differential equation}}$ and $\underbrace{f'(0) = 8, \; f(2) = 3e^2}_{\text{intial values}}$. Find $f(3)$.

$$f'(x) = \int f''(x)\,dx$$

$$= \int 3e^x + 4\,dx$$

$$= 3\int e^x\,dx + 4\int 1\,dx$$

$$= 3e^x + 4x + c_1$$

$$8 = f'(0) = 3e^0 + 4(0) + c_1$$

$$8 = 3 + c_1$$

$c_1 = 5$

$f'(x) = 3e^x + 4x + 5$

Repeat: $f(x) = \int f'(x)\,dx$

$= \int 3e^x + 4x + 5\,dx$

$= 3\int e^x dx + 4\int x\,dx + 5\int dx$

$= 3e^x + 4\frac{x^{1+1}}{1+1} + 5\frac{x^1}{1} + c_2$

$= 3e^x + 2x^2 + 5x + c_2$

$3e^2 = f(2) = 3e^2 + 2(2)^2 + 5(2) + c_2$

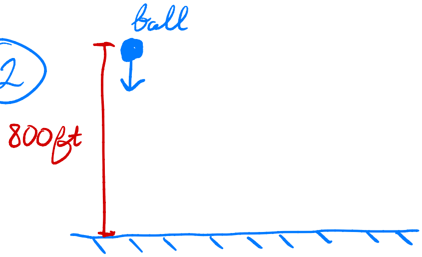
$0 = 8 + 10 + c_2$

$c_2 = -18$

$f(x) = 3e^x + 2x^2 + 5x - 18$

$f(3) = 3e^3 + 2(3)^2 + 5(3) - 18 = 3e^3 + 15$

2



Ball is dropped from 800ft in the air straight down and has an initial speed of 2 ft/sec. Use -32 ft/sec^2 for the acceleration due to gravity.

a) How long will it take for the ball to hit the ground?

$$a(t) = -32$$

- velocity = antiderive of acc.
- position = antiderivative of velocity.

$$v(t) = \int a(t)\,dt = \int -32\,dt = -32t + c_1$$

$$-2 = v(0) = -32(0) + c_1$$

$$c_1 = -2 \qquad \text{so} \qquad v(t) = -32t - 2$$

$$s(t) = \int v(t)\,dt = \int -32t - 2\,dt$$

$$= -32\int t\,dt - 2\int dt$$

$$= -32\,\frac{t^{1+1}}{1+1} - 2\,\frac{t^{0+1}}{0+1} + c_2$$

$$= -16t^2 - 2t + c_2$$

$$800 = s(0) = -16(0)^2 - 2(0) + c_2$$

$$c_2 = 800$$

$$s(t) = -16t^2 - 2t + 800$$

Ball on ground when $s(t)=0$.

$$0 = -16t^2 - 2t + 800$$
$$0 = 8t^2 + t - 400$$

$$t = \frac{-1 \pm \sqrt{1^2 - 4(8)(-400)}}{2(8)}$$
$$\approx \cancel{-7.134} \quad \text{and} \quad \approx \underline{7.009}$$

the ball hits the ground approximately 7.009 seconds after its dropped.

b) What is the velocity when the ball hits the ground?

At $t = 7.009$ the ball is on the ground, so

$$v(7.009) = -32(7.009) - 2 = -226.288 \text{ ft/sec}$$

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$$= \int \frac{1}{x} + 8e^x\,dx$$

$$= \int \frac{1}{x}\,dx + 8\int e^x\,dx$$

$$= \ln|x| + 8e^x + c$$

LON-CAPA
↓
$|x| = $ abs (x)

Recall: A function $F(x)$ is **an antiderivative** of $f(x)$ if $F'(x) = f(x)$.

We define the **indefinite integral** of $f(x)$ as

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① Let $\underbrace{f''(x) = 3e^x + 4}_{\text{ODE ordinary differential equation}}$ and $\underbrace{f'(0) = 8 \text{ and } f(2) = 3e^2}_{\text{initial values}}$. Find $f(3)$.

$$f'(x) = \int f''(x)\,dx = \int 3e^x + 4\,dx$$

$$= 3\int e^x\,dx + 4\int x^0\,dx$$

$$= 3e^x + 4\frac{x^{0+1}}{0+1} + c_1$$

$$= 3e^x + 4x + c_1$$

$8 = f'(0) = 3e^0 + 4(0) + c_1$

$8 = 3 + c_1$

$c_1 = 5$ So $f'(x) = 3e^x + 4x + 5$

Repeat: $f(x) = \int f'(x)\,dx = \int 3e^x + 4x + 5\,dx$

$= 3\int e^x dx + 4\int x\,dx + 5\int dx$

$= 3e^x + 4\dfrac{x^{1+1}}{1+1} + \dfrac{5x^{0+1}}{0+1} + c_2$

$= 3e^x + 2x^2 + 5x + c_2$

$3e^2 = f(2) = 3e^2 + 2(2)^2 + 5(2) + c_2$

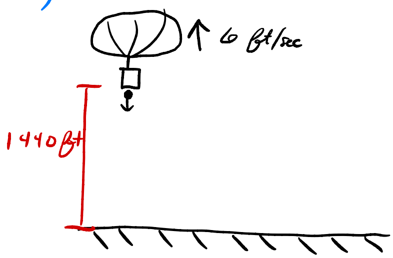
$0 = 8 + 10 + c_2$

$c_2 = -18$ So $f(x) = 3e^x + 2x^2 + 5x - 18$

$f(3) = 3e^3 + 2(3)^2 + 5(3) - 18 = 3e^3 + 15$

(2) A hot air Balloon is rising vertically at 6 ft/sec. A Ball is dropped from the hot air balloon 1440 ft above the ground. Use -32 ft/sec^2 as the acceleration due to gravity.

a) What time does the ball hit the ground?



$a(t) = -32$

- velocity = integral of acc.
- position = integral of velocity

$$v(t) = \int a(t)\,dt = \int -32\,dt = -32t + c_1$$

$$+6 = v(0) = -32(0) + c_1$$
$$c_1 = 6$$

$$v(t) = -32t + 6$$

$$s(t) = \int v(t)\,dt = \int -32t + 6\,dt$$
$$= -32\int t\,dt + 6\int dt$$
$$= -32\,\frac{t^{1+1}}{1+1} + 6\,\frac{t^{0+1}}{0+1} + c_2$$
$$= -16t^2 + 6t + c_2$$

$$1440 = s(0) = -16(0)^2 + 6(0) + c_2$$
$$c_2 = 1440$$

$$s(t) = -16t^2 + 6t + 1440$$

$$0 = -16t^2 + 6t + 1440$$

$$t = \frac{-6 \pm \sqrt{(6)^2 - 4(-16)(1440)}}{2(-16)}$$

$$\approx \cancel{-9.30} \quad \text{and} \quad \approx 9.68$$

9.68 seconds after the ball is dropped it hits the ground.

b) What is the velocity when the ball hits the ground?

@ $t = 9.68$ the ball is on the ground so the velocity at this instant is

$$v(9.68) = -32(9.68) + 6$$

$$= -303.76 \text{ ft/sec}$$

③ Rate of growth of a population of bacteria is

$$\frac{dP}{dt} = 8\sqrt{t}$$

P : population
t : time in days

initial pop is 400. Approximate the population after 7 days.

$$\left(P'(t) = \right) \frac{dP}{dt} = 8\sqrt{t} \qquad P(0) = 400$$

$$P(t) = \int \frac{dP}{dt}\, dt = \int 8\sqrt{t}\, dt$$

$$= 8 \int t^{1/2}\, dt$$

$$= 8 \frac{t^{1/2+1}}{1/2 + 1} + C$$

$$= 8 \cdot \frac{2}{3} t^{3/2} + C$$

$$400 = P(0) = \frac{16}{3} (0)^{3/2} + C$$

$$C = 400$$

$$P(t) = \frac{16}{3} t^{3/2} + 400$$

$$P(7) = \frac{16}{3} (7)^{3/2} + 400$$