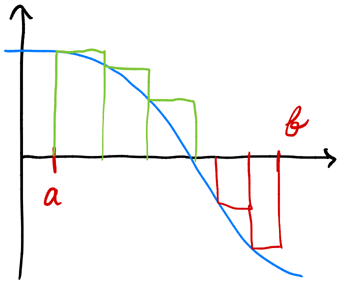
Lecture 29 : Definite integration I

Last time we approximat the singed area under a curve.



Signed area ≈ $\sum$ area of rect above
− $\sum$ area of rect below

Main idea : more rectangles = better approx.

$$\text{Signed Area of } f(x) = \lim_{n\to\infty} \underbrace{\sum_{i=1}^{n} f(x_i)\Delta x}_{\text{Right Riemann Sums}}$$

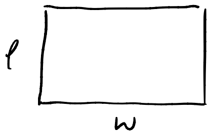
The definite integral of $f(x)$ on $[a, b]$ is the signed area under the curve $f(x)$ on $[a, b]$. It is denoted by

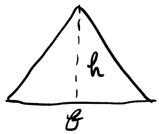
$$\int_a^b f(x)\,dx$$

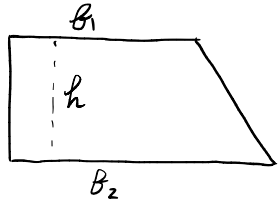
b: upper bound
a: lower bound

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

Recall:

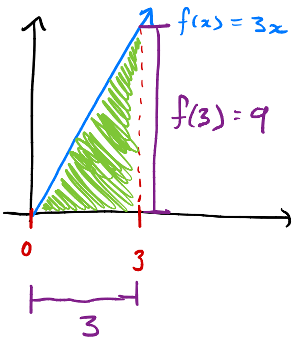






$A = lw$ $\qquad$ $A = \frac{1}{2}bh$ $\qquad$ $A = \frac{b_1 + b_2}{2}h$

e.g. ① Evaluate $\int_0^3 3x\,dx$ using shapes

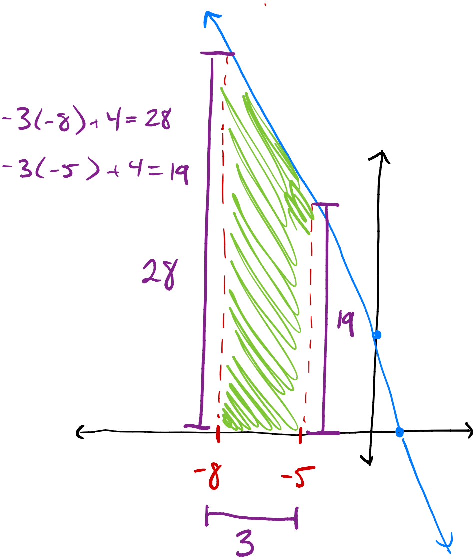


$\int_0^3 3x\,dx = \text{area}\Big($  $\Big)$

$= \frac{1}{2} \cdot 3 \cdot 9$

$= 27/2$

② Evaluate $\int_{-8}^{-5} -3x + 4\,dx$ using shapes.

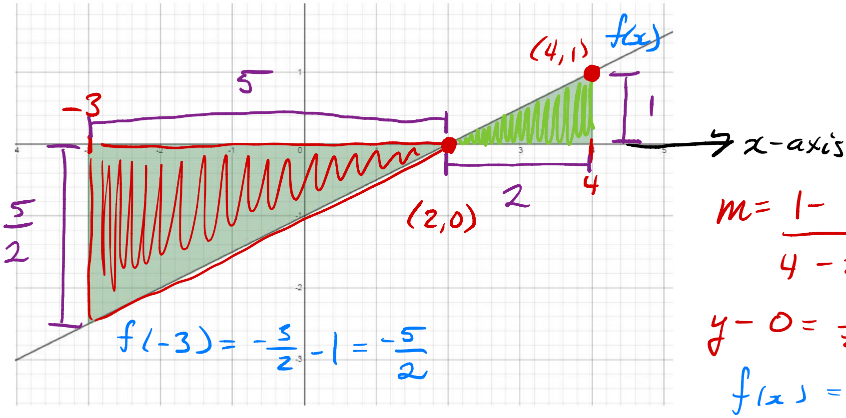


$\int_{-8}^{-5} -3x + 4\,dx = \text{area}\Big($  $\Big)$

$= \frac{(19+28)}{2} \cdot 3$

$= \frac{141}{3}$

③ Evaluate $\int_{-3}^{4} f(x)\,dx$



$m = \dfrac{1-0}{4-2} = \dfrac{1}{2}$

$y - 0 = \frac{1}{2}(x-2)$

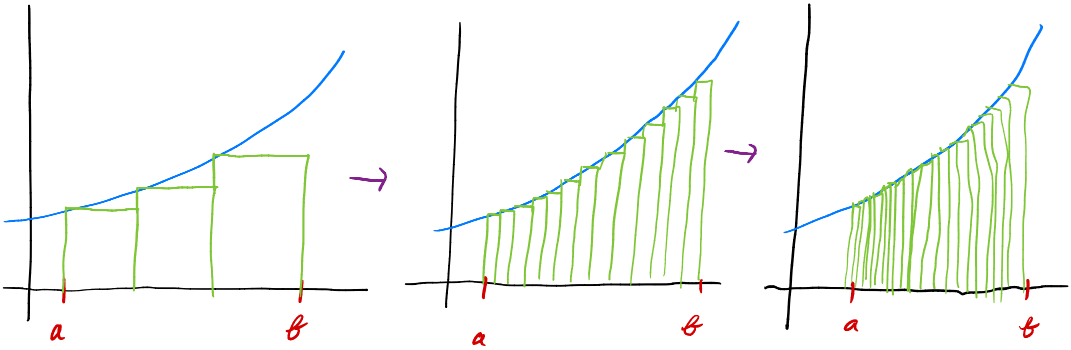
$f(x) = \frac{1}{2}x - 1$

$$\int_{-3}^{4} f(x)\,dx = \text{area}(\quad) - \text{area}(\quad)$$

$$= \frac{1}{2}(2)(1) \quad - \quad \frac{1}{2}(5)\left(\frac{5}{2}\right)$$

$$=$$

Lecture 29: Definite integral I



Question: what happens as the number of rectangles $n \to \infty$?

$$\text{Signed area of } f(x) = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \quad \left(= \lim_{n\to\infty} \sum_{i=0}^{n-1} f(x_i)\Delta x \right)$$

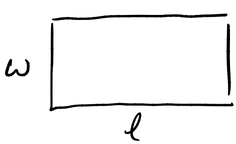
The definite integral of $f(x)$ on $[a,b]$ is the signed area of $f(x)$ on $[a,b]$

$$\int_a^b f(x)\,dx .$$

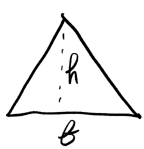
b: upper bound
a: lower bound

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

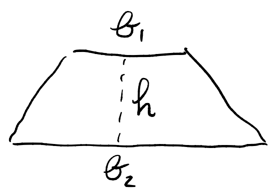
Recall:



$A = lw$



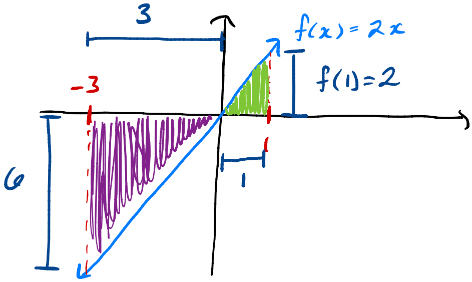
$A = \frac{1}{2} bh$

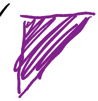


$A = \dfrac{b_1 + b_2}{2} \cdot h$

e.g.

1. Evaluate $\int_{-3}^{1} 2x\,dx$ using shapes.



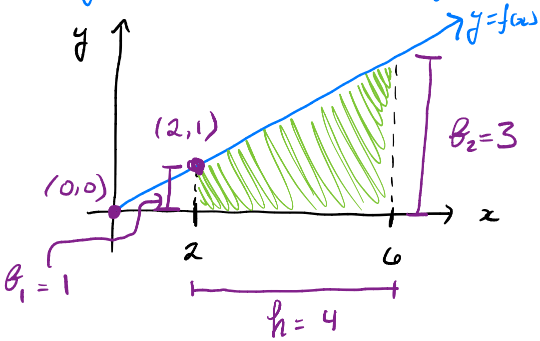
$\int_{-3}^{1} 2x\,dx$ = area() − area()

$= \frac{1}{2}(1)(2) - \frac{1}{2}(3)(6)$

$= 1 - 9$

$= -8$

2. Write an integral that represents the shaded region. Compute integral.

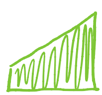


$\int_{2}^{6} f(x)\,dx$

slope $m = \frac{1-0}{2-0} = \frac{1}{2}$

$y - 0 = \frac{1}{2}(x - 0)$

$y = \frac{1}{2}x$

$\int_{2}^{6} \frac{1}{2}x\,dx$ = area() $= \frac{(1+3)}{2} 4 = 8$