Lecture 30: Definite integration II

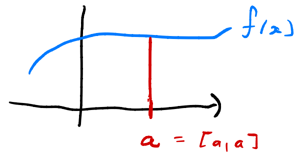
Recall: We defined the definite integral by

$$\int_a^b f(x)\,dx := \underline{\text{signed area}} \text{ between } f(x) \text{ and the } x\text{-axis on } [a, b].$$

Properties of the definite integral

Let $a, b, c,$ and k be constants (just numbers)

$$\int_a^a f(x)\,dx = 0$$



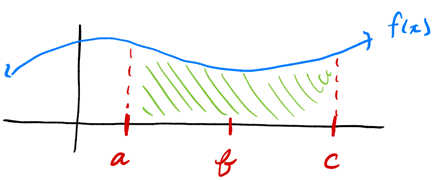
$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

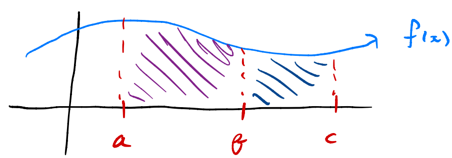
$$\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

definite integral is a linear operation

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$





Warning: $\displaystyle\int_a^b f(x)\,g(x)\,dx \neq \int_a^b f(x)\,dx \cdot \int_a^b g(x)\,dx$

e.g. ① Let $\int_5^9 2x^3\,dx = 2968$.
Compute $\int_9^5 2x^3\,dx$ and $\int_5^9 20x^3\,dx$.

$$\int_9^5 2x^3\,dx = -\int_5^9 2x^3\,dx = -2968$$

$$\int_5^9 20x^3\,dx = \int_5^9 10\cdot 2x^3\,dx = 10\int_5^9 2x^3\,dx = 29680$$

② Let $\int_1^2 x^2\,dx = 7/3$, $\int_1^2 x\,dx = 3/2$, $\int_1^2 dx = 1$

Compute $\int_1^2 (-4x^2 + 3x - 3)\,dx$

$$\begin{aligned}\int_1^2 (-4x^2 + 3x - 3)\,dx &= -4\int_1^2 x^2\,dx + 3\int_1^2 x\,dx - 3\int_1^2 dx \\ &= -4(7/3) + 3(3/2) - 3(1) \\ &= -28/3 + 9/2 - 3\end{aligned}$$

③ If $\int_{-5}^{14} h(t)\,dt = 28$, $\int_4^{14} h(t)\,dt = 36$, then find $\int_{-5}^{4} h(t)\,dt$.

$$\begin{aligned}\int_{-5}^{4} h(t)\,dt &= \int_{-5}^{14} h(t)\,dt + \int_{14}^{4} h(t)\,dt \\ &= \int_{-5}^{14} h(t)\,dt - \int_{4}^{14} h(t)\,dt \\ &= 28 - 36 \\ &= -8\end{aligned}$$

(4) If $\int_5^7 f(x)\,dx = 1$, $\int_7^{12} f(x)\,dx = 5$, $\int_5^{12} g(x)\,dx = 14$ then compute $\int_5^{12} 7f(x) - 6g(x)\,dx$

$$\int_5^{12} 7f(x) - 6g(x)\,dx = 7\int_5^{12} f(x)\,dx - 6\int_5^{12} g(x)\,dx$$

$$= 7\left(\int_5^7 f(x)\,dx + \int_7^{12} f(x)\,dx\right) - 6\int_5^{12} g(x)\,dx$$

$$= 7(1 + 5) - 6(14)$$

$$= -42$$

(5) Let $\int_a^b g(x)\,dx = 2$, $\int_a^c g(x)\,dx = 3\int_a^b g(x)\,dx$

Find $\int_b^c g(x)\,dx$.

$\int$

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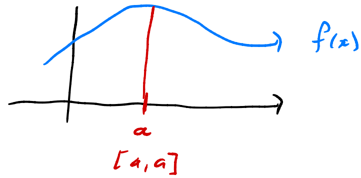

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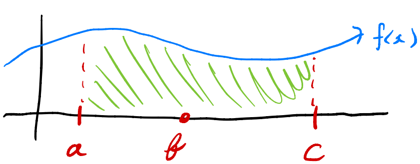
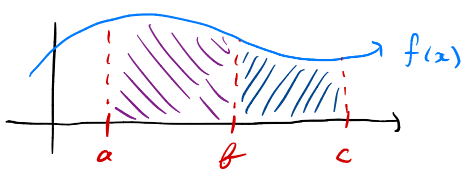
* note b need not be in $[a,c]$ *





Warning: $\int_a^b f(x)g(x)\,dx \neq \int_a^b f(x)\,dx \int_a^b g(x)\,dx$

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(5) Let $\int_a^b g(x)\,dx = 2$ and $\int_a^c g(x)\,dx = 3\int_a^b g(x)\,dx$

Find $\int_b^c g(x)\,dx$.

$$\int_b^c g(x)\,dx = \int_b^a g(x)\,dx + \int_a^c g(x)\,dx$$

$$= -\int_a^b g(x)\,dx + 3\int_a^b g(x)\,dx$$

$$= -2 + 3(2)$$

$$= 4$$