Lecture 32: The fundamental theorem of calculus II

Recall: Let $f(x)$ be a function with an antiderivative $F(x)$, then

Fundamental Theorem of Calculus: $\int_a^b f(x)\,dx = F(x)\Big|_a^b$

e.g. ① The growth rate of a population of a country is

$$P(t) = \sqrt{t}\,(2370t + 6270)$$

where t is time in years.
How much does the population increase from year 1 to year 4?

"Integral from a to b is the net change from a to b"

$$\int t^n\,dt = \frac{t^{n+1}}{n+1} + C$$

$$\int_1^4 \sqrt{t}\,(2370t + 6270)\,dt$$

$$= \int_1^4 2370t^{3/2} + 6270t^{1/2}\,dt$$

$$\overset{FTC}{=} \left(2370 \cdot \tfrac{2}{5}t^{5/2} + 6270 \cdot \tfrac{2}{3}t^{3/2}\right)\Big|_1^4$$

$$= 2370 \cdot \tfrac{2}{5}(4)^{5/2} + 6270 \cdot \tfrac{2}{3}(4)^{3/2} - \left[2370 \cdot \tfrac{2}{5}(1)^{5/2} + 6270 \cdot \tfrac{2}{3}(1)^{3/2}\right]$$

$$= 58648$$

② The velocity function in meters/min, of a particle moving in a straight line is

$$v(t) = 5t - 3$$

where t is time in minutes.

a) Find the $\overbrace{\text{displacement}}^{=\text{ net change of position.}}$ of the particle from $t=2$ to $t=5$.

$$\text{Displacement} = \int_2^5 s'(t)\,dt$$

$$= \int_2^5 v(t)\,dt$$

$$= \int_2^5 5t - 3\,dt$$

$$\overset{\text{FTC}}{=} \left(\frac{5}{2}t^2 - 3t\right)\Big|_2^5$$

$$= \frac{5}{2}(5)^2 - 3(5) - \left[\frac{5}{2}(2)^2 - 3(2)\right]$$

$$= 43.5 \text{ meters.}$$

b) Find time x when the displacement is zero after the particle starts moving.

Displacement from $t=0$ to $t=x$:

$$\int_0^x 5t - 3\,dt = 0 \quad \text{solve for } x$$

$$\int_0^x 5t - 3\,dt \overset{\text{FTC}}{=} \left(\frac{5}{2}t^2 - 3t\right)\Big|_0^x$$

$$= \frac{5}{2}x^2 - 3x - \left[\frac{5}{2}(0)^2 - 3(0)\right]$$

$$= \frac{5}{2}x^2 - 3x$$

$$\frac{5}{2}x^2 - 3x = 0 \quad \text{Solve for } x$$

$$\frac{5}{2}x\left(x - \frac{6}{5}\right) = 0$$

$\cancel{x = 0}$ uninteresting or $\underbrace{x = 6/5 \text{ minutes.}}$

Lecture 32: The fundamental theorem of calculus II

Recall: Let $f(x)$ be a function with an antiderivative $F(x)$, then

Fundamental Theorem of Calculus:

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b$$

Main idea:

Net change of $f(x)$ from a to b is

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

e.g. ① The growth rate of a population of a country is

$$R(t) = \sqrt{t}\,(2370t + 6270)$$

where t is time in years. How much does the population increase from year 1 to year 4?

$\underbrace{\text{population increase}}_{\text{net change of population from } t=1 \text{ to } t=4.}$

$$\int t^n\,dt = \frac{t^{n+1}}{n+1}$$

$$\int_1^4 \sqrt{t}\,(2370t + 6270)\,dt = \int_1^4 2370t^{3/2} + 6270t^{1/2}\,dt$$

$$\overset{FTC}{=} \left(2370 \cdot \frac{2}{5} t^{5/2} + 6270 \cdot \frac{2}{3} t^{3/2}\right)\Big|_1^4$$

$$= 2370 \cdot \frac{2}{5}(4)^{5/2} + 6270 \cdot \frac{2}{3}(4)^{3/2} - \left[2370 \cdot \frac{2}{5}(1)^{5/2} + 6270 \cdot \frac{2}{3}(1)^{3/2}\right]$$

$= 58648$ people.

(2) The velocity function, in meters/min, of a particle moving in a straight line is

$$v(t) = 5t - 3 \leftarrow = s'(t)$$

where t is time in minutes.

a) Find the displacement (net change of position) of the particle from $t = 2$ to $t = 5$.

$$\int_2^5 s'(t)\,dt = \int_2^5 v(t)\,dt = \int_2^5 5t - 3\,dt$$

$$\overset{FTC}{=} \left(\frac{5}{2}t^2 - 3t + c\right)\Big|_2^5 = 43.5 \text{ meters.}$$

$$= \frac{5}{2}(5)^2 - 3(5) + c - \left[\frac{5}{2}(2)^2 - 3(2) + c\right]$$

$$= \frac{5}{2}(5)^2 - 3(5) - \left[\frac{5}{2}(2)^2 - 3(2)\right] + c - c$$

$$= \left(\frac{5}{2}t^2 - 3t\right)\Big|_2^5$$

b) Find the time x when the displacement is zero after the particle starts moving.

$$\int_0^x 5t - 3\,dt = 0 \quad \text{solve for } x.$$

$$\int_0^x 5t - 3\,dt = \left(\frac{5}{2}t^2 - 3t\right)\Big|_0^x$$

$$= \frac{5}{2}x^2 - 3x - \left[\frac{5}{2}(0)^2 - 3(0)\right]$$

$$= \frac{5}{2}x^2 - 3x = \frac{5}{2}x\left(x - 6/5\right)$$