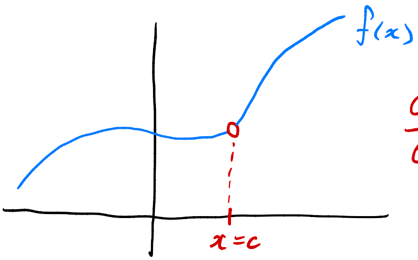
Lecture 4: Continuity

A function $f(x)$ is **continuous** at $x=c$ if there are no breaks / holes

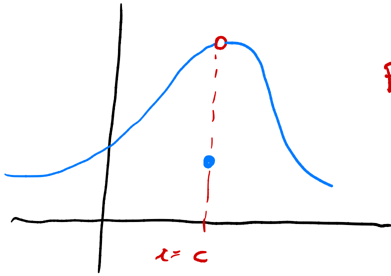
Types of Discontinuity

Hole



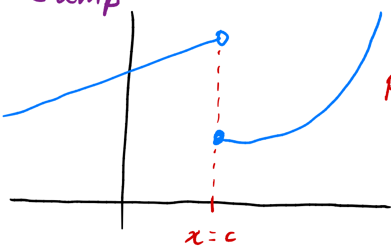
$\frac{0}{0}$

Hole (removable discontinuity)



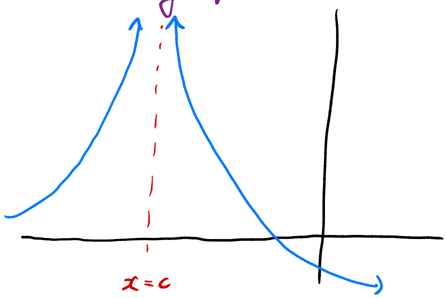
piecewise

Jump



piecewise

Asymptote



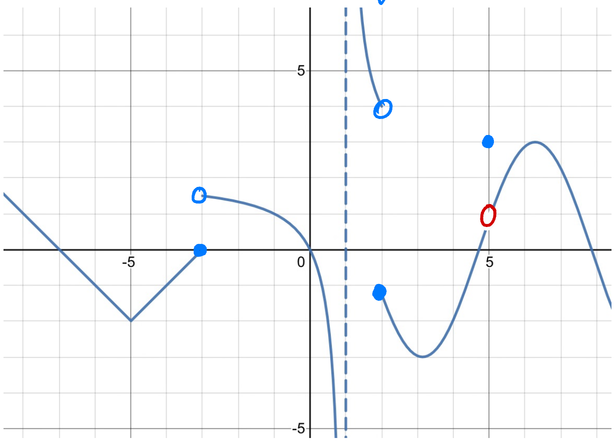
$\frac{0}{0}$ or $\frac{1}{0}$

Question: When is $f(x)$ continuous at $x=c$?

1) $f(c)$ to be defined
2) $\lim_{x \to c} f(x)$ exists
3) $f(c) = \lim_{x \to c} f(x)$

need <u>all</u> of these

e.g. ① find/classify all discont. of f



$x = -3$; Jump
1) $f(-3) = 0$ ✓
2) $\lim_{x\to -3} f(x)$ DNE ✗

$x = 1$; asymptote

$x = 2$; Jump

$x = 5$; hole
1) $f(5) = 3$ ✓
2) $\lim_{x\to 5} f(x) = 1$ ✓
3) $f(5) = 3 \neq 1 = \lim_{x\to 5} f(x)$ ✗

② $f(x) = \dfrac{x^2+2x-3}{x^2+5x-6}$ find/classify all discontinuities

$$= \frac{(x+3)(x-1)}{(x+6)(x-1)}$$

find all $x=c$ such that $f(c) = \frac{0}{0}$ or $\frac{1}{0}$

all discont.

$x = 1$: $f(1) = \frac{0}{0}$;

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{(x+3)\cancel{(x-1)}}{(x+6)\cancel{(x-1)}} = \lim_{x\to 1} \frac{x+3}{x+6} = \frac{1+3}{1+6} = \frac{4}{7}$$

Since limit is finite then HOLE!

$x = -6$: $f(-6) = \dfrac{\text{nonzero}}{0}$ asymptote

③ $f(x) = \begin{cases} -8x - \pi/2 & x \leq -\pi/2 \\ \cos x & -\pi/2 < x < \pi/2 \\ 9\sin x + 3 & x \geq \pi/2 \end{cases}$

find / classify all discont.

$-8x-\pi/2$, $\cos x$, $9\sin x + 3$ are all continuous

possible discont. at $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$

$x = -\pi/2$:

$$\lim_{x \to -\frac{\pi}{2}^-} f(x) = \lim_{x \to -\frac{\pi}{2}^-} \left(-8x - \frac{\pi}{2}\right) = \frac{7\pi}{2}$$

$$\lim_{x \to -\frac{\pi}{2}^+} f(x) = \lim_{x \to -\frac{\pi}{2}^+} \cos x = 0$$

jump

$\lim_{x \to -\frac{\pi}{2}} f(x)$ DNE so we fail (2) of continuity def.

we have a jump.

Lecture 4: Continuity

HW3 #10 $f(x) = \begin{cases} -7x - \pi/2 & x \le -\pi/2 \\ \cos x & -\pi/2 < x < \pi/2 \\ 2\sin x + 2 & x \ge \pi/2 \end{cases}$

$\lim_{x \to \frac{\pi}{2}} f(x) = ?$

$$\lim_{x \to \pi/2^-} f(x) = \lim_{x \to \pi/2^-} \cos x = \cos\left(\frac{\pi}{2}\right) = 0$$

$$\neq$$

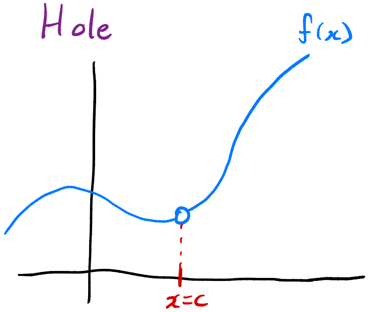
$$\lim_{x \to \pi/2^+} f(x) = \lim_{x \to \pi/2^+} (2\sin x + 2) = 4$$

$\lim_{x \to \pi/2} f(x)$ DNE.

A function $f(x)$ is continuous at $x = c$ if there are no breaks/holes.

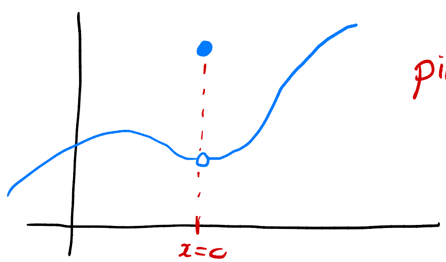
Types of Discontinuities

Hole



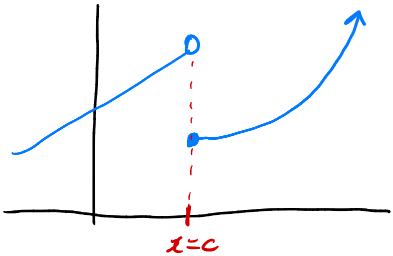
$\frac{0}{0}$

Hole (removable)



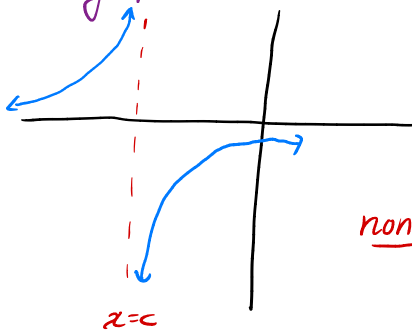
piecewise

Jump



piecewise

Asymptote



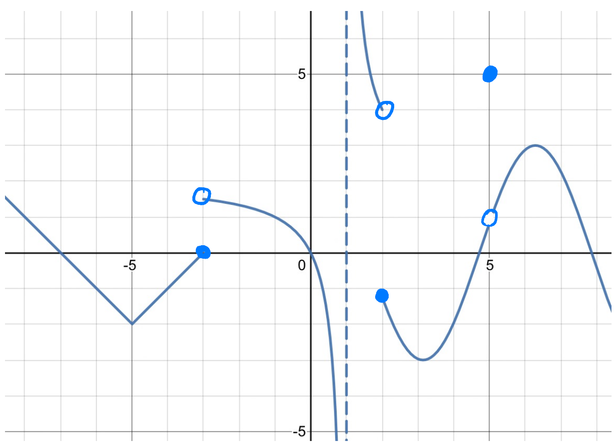
$\frac{\text{nonzero}}{0}$ or $\frac{0}{0}$

Question: When is $f(x)$ continuous at $x=c$?

1) $f(c)$ must be defined
2) $\lim_{x \to c} f(x)$ must exist
3) $f(c) = \lim_{x \to c} f(x)$

need <u>all</u> of these to be continuous

e.g. ① find/classify all discont.



		Fail
$x = 1$;	asymptote	(1)
$x = -3$;	jump	(2) (3)
$x = 2$;	jump	(2) (3)
$x = 5$;	hole	(3)

② $f(x) = \dfrac{x^2 + 2x - 3}{x^2 + 5x - 6}$ find/classify all discont.

$= \dfrac{(x+3)(x-1)}{(x+6)(x-1)}$

We want to find all $x=c$ such that $f(c) = \frac{0}{0}$ or $\frac{\text{nonzero}}{0}$

all discont.

$x = -6$: $f(-6) = \dfrac{(-3)(-7)}{0} = \dfrac{21}{0}$ asymptote

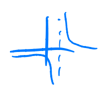
$x = 1$: $f(1) = \dfrac{0}{0}$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{(x+3)\cancel{(x-1)}}{(x+6)\cancel{(x-1)}}$$

$$= \lim_{x \to 1} \frac{x+3}{x+6} = \frac{1+3}{1+6} = \underline{\frac{4}{7}}$$

HOLE!

$g(x) = \dfrac{(x-1)}{(x-1)^2}$ $\quad g(1) = \dfrac{0}{0}$ $\quad \displaystyle\lim_{x \to 1} \frac{x-1}{(x-1)^2} = \lim_{x \to 1} \frac{1}{x-1} = \text{DNE}$



③ $f(x) = \begin{cases} -8x - \pi/2 & x \leqslant -\pi/2 \\ \cos x & -\pi/2 < x < \pi/2 \\ 9\sin x + 3 & x \geqslant \pi/2 \end{cases}$

find / classify all discontinuities of f.

potential discontinuities at $x = -\frac{\pi}{2}$ or $\frac{\pi}{2}$

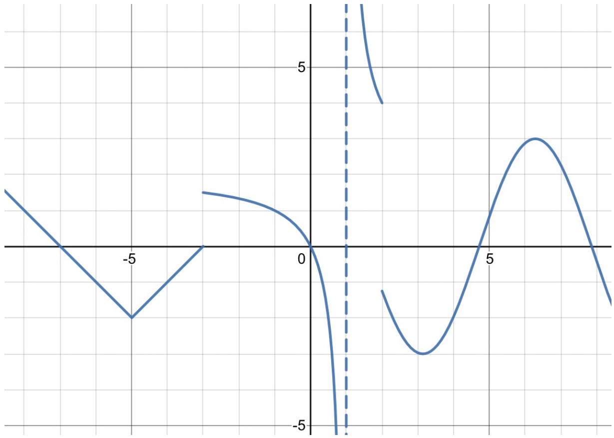
$x = -\frac{\pi}{2}$: $\lim_{x \to -\frac{\pi}{2}^-} f(x) = \lim_{x \to -\frac{\pi}{2}^-} \left(-8x - \frac{\pi}{2}\right) = \frac{7\pi}{2}$

$\neq$

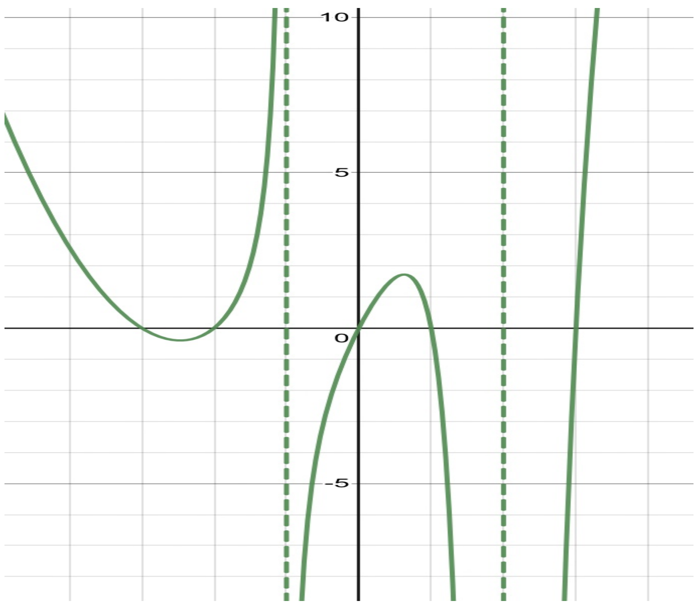
$\lim_{x \to -\frac{\pi}{2}^+} f(x) = \lim_{x \to -\frac{\pi}{2}^+} (\cos x) = 0$

$\lim_{x \to \pi/2} f(x)$ DNE

So $x = -\pi/2$ is a discont (fails (2))



5
-5
0
5
-5



10
5
0
-5